1. Find the derivatives of the following functions. You do **not** need to simplify your solutions.

   $g(x) = \dfrac{1}{\sqrt[3]{x+\sqrt{x}}}$

Solution: On this one, it's best to view the function as $g(x) = \left(x+\sqrt{x}\right)^{-1/3}$.

$$g'(x) = -\tfrac{1}{3}\left(x+\sqrt{x}\right)^{-4/3}\left(1+\tfrac{1}{2\sqrt{x}}\right)$$

2. Consider the graph of $y = 10x^3 - x^5$.

[4 points] (a) List the intervals on which this function is increasing.

Solution: The derivative is

$$y' = 30x^2 - 5x^4 = 5x^2(6 - x^2).$$

So, the critical points are $x = \pm\sqrt{6}$ and $x = 0$. We then check that y' is negative when $x < -\sqrt{6}$ or $x > \sqrt{6}$, and y' is positive when $-\sqrt{6} < x < 0$ or $0 < x < \sqrt{6}$. So, y is increasing on the intervals $(-\sqrt{6}, 0)$ and $(0, \sqrt{6})$. (It's decreasing on $(-\infty, \sqrt{6})$ and $(\sqrt{6}, \infty)$.)

[1 point] (b) Give the x-coordinates of all local minima, or say that there are none.

[1 point] (c) Give the x-coordinates of all local maxima, or say that there are none.

Solution: By the first derivative test, there is a local minimum at $x = -\sqrt{6}$ and a local maximum at $x = \sqrt{6}$.

[4 points] (d) List the intervals on which this function is concave up.

Solution: First, we find the second derivative:

$$y'' = 60x - 20x^3 = 20x(3 - x^2).$$

So, $y'' = 0$ at $x = \pm\sqrt{3}$ and at $x = 0$. We then figure out that y'' is positive on $(-\infty, -\sqrt{3})$, negative on $(-\sqrt{3}, 0)$, positive on $(0, \sqrt{3})$, and negative on $(\sqrt{3}, \infty)$. So, y is concave up on $(-\infty, -\sqrt{3})$ and on $(0, \sqrt{3})$.

[2 points] (e) List the x-coordinates of all inflection points.

Solution: There are inflection points where y'' changes signs. From the previous question, this happens at $x = \pm\sqrt{3}$ and at $x = 0$.

[8 points] 3. (a) Find the slope of the tangent line to the curve defined by

$$x^2y + y^4 = 4 + 2x$$

at the point $(-1, 1)$.

Solution: Using implicit differentiation,

$$2xy + x^2\frac{dy}{dx} + 4y^3\frac{dy}{dx} = 2.$$

Solving for $\frac{dy}{dx}$, we get

$$\frac{dy}{dx} = \frac{2 - 2xy}{x^2 + 4y^3}.$$

At $(-1, 1)$, this is $4/5$.

[8 points] (b) Using linear approximation, estimate a y-value on this curve when $x = -0.9$.

Solution: We know that the curve contains the point $(-1, 1)$, at which it has slope $4/5$. The change from $x = -1$ to $x = -0.9$ is 0.1. So. the linear approximation for the y-value of the curve at $x = -0.9$ is

$$1 + (0.1)\tfrac{4}{5} = 1.08.$$

[12 points] 4. Find the minimum and maximum value taken by the function $f(x) = 2\sqrt{x} - x$ in the interval $[0, 3]$.

Solution: First, we find all critical points in the interval:

$$f'(x) = \frac{1}{\sqrt{x}} - 1 = 0,$$

which has solution $x = 1$. So, the maximum and minimum on $[0, 3]$ must occur either at $x = 1$ or at one of the endpoints $x = 0$ and $x = 3$. Evaluating, we find

$$f(0) = 0$$
$$f(1) = 1$$
$$f(3) = 2\sqrt{3} - 3 \approx 0.464.$$

So, the minimum of the function is 0 and the maximum is 1.