

*I pledge that I have neither given nor received
unauthorized assistance during this examination.*

Signature:

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise, **you must show your work** sufficiently much that it's clear how you arrived at your answer.
- There are 10 problems on 11 pages.

Question	Points	Score
1	8	
2	8	
3	12	
4	9	
5	9	
6	11	
7	10	
8	11	
9	12	
10	10	
Total:	100	

Good luck!

[8 points] 1. Find the derivatives of the following functions. Please do **not** simplify your solutions.

(a) $f(x) = e^{x^3} \sin x$

(b) $g(t) = \frac{1}{3t^2} - \sqrt{t} + \ln(4t + 1) + 1$

[8 points] 2. Let $f(x) = \frac{x}{1+x^2}$. Find an equation for the tangent line to the graph of $f(x)$ at $x = 2$.

[12 points] 3. Compute the following integrals.

(a) $\int \left(x^2 + \frac{2}{x} \right) dx$

(b) $\int 2e^{\sin t} \cos t \, dt$

(c) $\int_0^2 3e^{-2x} \, dx$

- [9 points] 4. Compute the following limits. If the limit does not exist, say so. Justify your answers by showing work or otherwise explaining your reasoning.

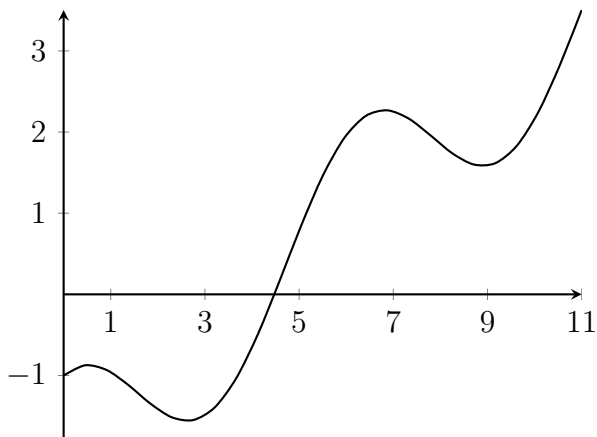
(a) $\lim_{x \rightarrow -\infty} \frac{3x^4 + x^3 - 100x^2 + 1}{5x^4 + 2}$

(b) $\lim_{x \rightarrow \infty} x e^{-2x}$

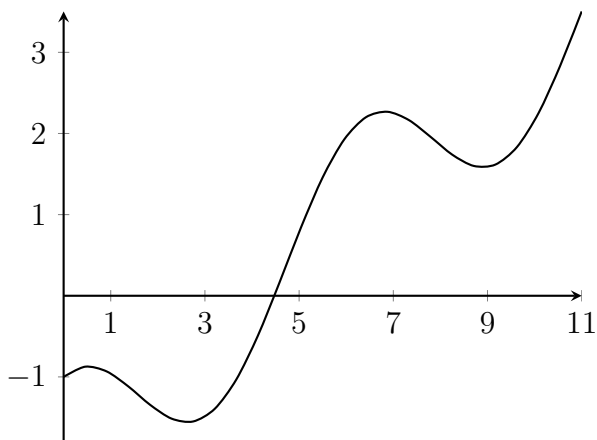
(c) $\lim_{x \rightarrow 3} \frac{2 \sin(x - 3)}{\cos(x - 3)}$

[9 points] 5. This question asks about you to label certain points on a graph of a function with domain $[0, 11]$, shown in the problems below.

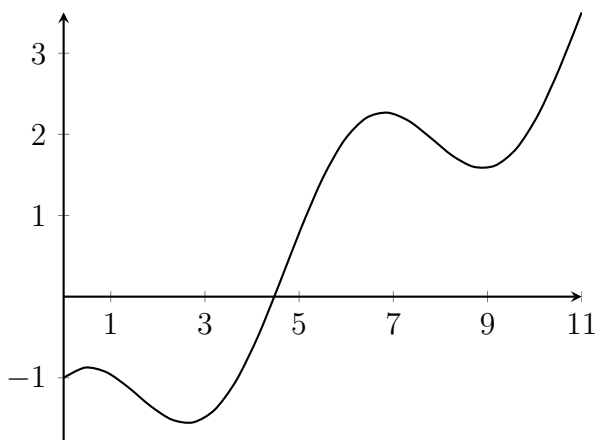
- (a) Draw points on the graph at the (x, y) locations where **local** maxima or minima occur.



- (b) Draw points on the graph at the (x, y) locations where **absolute** maxima or minima occur for the function on the domain $[0, 11]$.



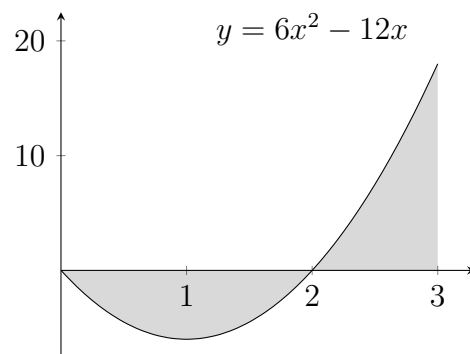
- (c) Draw points on the graph at the (x, y) locations where inflection points occur.



- [11 points] 6. Find the two nonnegative numbers x and y that add up to 6 and make xy^2 as large as possible.

- [10 points] 7. A spherical balloon is filled with air at a rate of 2 cm^3 per minute. When the balloon has radius 8 cm, what is the rate of change of its radius? (The volume of a sphere with radius r is given by the formula $V = \frac{4}{3}\pi r^3$.)

- [11 points] 8. Compute the total (unsigned) area between the graph of $f(x) = 6x^2 - 12x$ and the x -axis from $x = 0$ to $x = 3$. This is the shaded region in the graph sketched here.



[12 points] 9. Let $f(x) = x^4 + 4x^3 - 16x + 1$. The first and second derivatives of f are

$$f'(x) = 4x^3 + 12x^2 - 16 = 4(x+2)^2(x-1)$$

and

$$f''(x) = 12x^2 + 24x = 12x(x+2).$$

(a) State all intervals where $f(x)$ is increasing. If there are none, write *none*.

(b) State all intervals where $f'(x)$ is concave up. If there are none, write *none*.

(c) Give all x -values where local minima occur. If there are none, write *none*.

(d) Give all x -values where local maxima occur. If there are none, write *none*.

(e) Give all x -values where inflection points occur. If there are none, write *none*.

[10 points] 10. Consider the curve defined by the equation

$$e^{xy} = 1 + 2y.$$

Find $\frac{dy}{dx}$. Your answer will be in terms of both x and y .