Name: _____

Note: When transforming expressions into equivalent forms, write a chain of equalities. Don't just write one expression after the next without stating the relationship between the expressions.

When solving equations, write down one equation after another. Do not cross out or write on top of an equation.

1. Find all solutions to $|3x + 1| = 5$.

Solution: We want to solve $3x + 1 = 5$ and $3x + 1 = -5$. From the first, we get

$$3x = 4$$

and then $x = \frac{4}{3}$. From the second, we get

$$3x = 6$$

and then $x = 2$. So the two solutions are $x = \frac{4}{3}$ and $x = 2$.

2. Give all solutions to $(x-5)^2(x+3) \leq 0$. (You can express your answer in any form you'd like, so long as it's clear what you mean.)

Solution: The zeros of $(x-5)^2(x+3)$ occur at $x = -3$ and $x = 5$. So we need to check the behavior of $(x-5)^2(x+3)$ on $(-\infty, -3)$, $(-3, 5)$, and $(5, \infty)$.

- On $(-\infty, -3)$, the factor $(x-5)^2$ is positive and $(x+3)$ is negative. So $(x-5)^2(x+3)$ is negative here.
- On $(-3, 5)$, the factor $(x-5)^2$ is positive and $(x+3)$ is positive. So $(x-5)^2(x+3)$ is positive here.
- On $(5, \infty)$, the factor $(x-5)^2$ is positive and $(x+3)$ is positive. So $(x-5)^2(x+3)$ is positive here.

All together, $(x-5)^2(x+3)$ is negative for $x < -3$. And it's equal to zero at $x = -3$ and also $x = 5$. So the final answer in interval notation is $(-\infty, -3] \cup \{5\}$.

3. A rectangular garden has area 2400 sq. ft. and is surrounded by 200 ft of fence. What are its dimensions?

Solution: Let x and y be the length and width of the fence. We know that $xy = 2400$ and $2x + 2y = 200$. From the second equation, we get $y = 100 - x$. Substituting into the first equation, we get

$$x(100 - x) = 2400.$$

We expand and subtract 2400 from both sides to get

$$100x - x^2 - 2400 = 0.$$

We multiply both sides by -1 to get

$$x^2 - 100x + 2400 = 0.$$

You can factor this as

$$(x - 40)(x - 60) = 0.$$

So there are two solutions, $x = 40$ and $x = 60$. The first solution gives $y = 60$ and the second $y = 40$. So the dimensions in either case are 40×60 ft.