• DON’T PANIC! If you get stuck, take a deep breath and go on to the next question.

• Unless the problem says otherwise you must show your work sufficiently much that it’s clear to me how you arrived at your answer.

• You may use a graphing or scientific calculator on this exam. You may not use a phone.

• You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.

• There are 9 problems on 9 pages.

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18</td>
<td></td>
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<tr>
<td>2</td>
<td>9</td>
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<td></td>
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<td><strong>Total:</strong></td>
<td><strong>106</strong></td>
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</tbody>
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Good luck!
1. Find all solutions to the following equations and inequalities. For inequalities with a range of solutions, you can give your answer using any notation (e.g., you may say that the solution set is \([5, 6)\) or that it’s given by \(5 \leq x < 6\)). If no solutions exist, say so.

(a) \(4x - 7 = 8 + 9x\)

**Solution:** \(x = -3\)

(b) \(x^2 + 9x + 20 = 0\)

**Solution:** \(x = -5, -4\)

(c) \(\sqrt{2x + 1} = x - 1\)

**Solution:** Squaring both sides, simplifying, and factoring gives the equation \(x(x - 4) = 0\), with solutions \(x = 0\) and \(x = 4\). But of these two solutions, only \(x = 4\) solves the original equation.
(d) \(-x + 3 > 4x\)

Solution: \(x < 3/5\), or \((-\infty, \frac{3}{5})\)

(e) \((x + 3)^2(x + 1) > 0\)

Solution: \((x + 3)^2\) is always positive, and \(x + 1\) is positive when \(x > -1\) and negative when \(x < -1\). So the whole expression is positive when \(x > -1\), or on the interval \((-1, \infty)\).

(f) \(x^5 = -2x^4\)

Solution: Shifting and factoring gives \(x^4(x + 2) = 0\), yielding solutions \(x = 0\) and \(x = -2\).
2. Simplify the following expressions as much as possible. You may assume that \( x \neq 0 \), so that expressions like \( \frac{1}{x} \) are well defined.

(a) \( \frac{x^2}{\sqrt{xx^{-3}}} \)

Solution:
\[
\frac{x^2}{\sqrt{xx^{-3}}} = x^2 x^3 x^{-1/2} = x^{2+3-1/2} = x^{9/2}
\]

(b) \( (x^2 x^3)^4 \)

Solution:
\[
(x^2 x^3)^4 = (x^5)^4 = x^{5(4)} = x^{20}
\]

(c) \( (x^{1/2} + x^{-1/2})^2 \)

Solution:
\[
(x^{1/2} + x^{-1/2})^2 = (x^{1/2})^2 + 2x^{1/2}x^{-1/2} + (x^{-1/2})^2 = x + 2 + \frac{1}{x}
\]
3. What is the domain of the function \( f(x) = \frac{1}{x-1} + \frac{x^2}{x+1} \)?

**Solution:** The domain is everything that can be plugged in, which is all real numbers except for 1 and \(-1\) (which would make a denominator zero).

4. Consider the following graph:

![Graph](image)

(a) Is this the graph of a function?

**Solution:** Yes.

(b) If the answer to part (a) is yes, what is the domain of the function? Give your answer in interval notation.

**Solution:** \([2, 7]\)

(c) If the answer to part (b) is yes, what is the range of the function? Give your answer in interval notation.

**Solution:** \([1, 5]\)
5. Let \( f(x) = -x^2 + 10x \).

(a) \[6\text{ points}\] Express \( f \) in standard form.

**Solution:**

\[
f(x) = -(x^2 - 10x) = -(x^2 - 10x + 25) + 25 = -(x - 5)^2 + 25
\]

(b) \[3\text{ points}\] What is the vertex of \( f \)?

**Solution:** From the standard form, we see that it’s \((5, 25)\)

(c) \[3\text{ points}\] List all \( x \)-intercepts of \( f \).

**Solution:** The \( x \)-intercepts are the roots of \( f(x) \), which we find by solving \( f(x) = 0 \):

\[
0 = -x^2 + 10x = -x(x - 10),
\]

so the roots are 0 and 10.

(d) \[3\text{ points}\] List all \( y \)-intercepts of \( f \).

**Solution:** The \( y \)-intercept of \( f(x) \) is found by plugging 0 in for \( x \):

\[
f(0) = 0
\]

So the \( y \)-intercept is 0.

(e) \[3\text{ points}\] Sketch the graph \( y = f(x) \) on these axes:
6. Expand and simplify the following expressions:

(a) \((5x + 1)^2\)

\[
\text{Solution: } 25x^2 + 10x + 1
\]

(b) \((y - 1)(y^2 + y + 4)\)

\[
\text{Solution: } y^3 + 3y - 4
\]
7. Here is the graph of the function $f(x)$ on the domain $[0, 3]$:

Sketch the graphs of the following functions on the axes below.

(a) $f(-x)$
(b) $f(x)/2$
(c) $-f(x + 1)$
8. A small-appliance manufacturing company finds that the production of \( x \) toaster ovens per month leads to monthly production costs of \( y \) dollars, with the relationship between \( x \) and \( y \) graphed below:

![Graph showing the relationship between production (x) and costs (y).](image)

(a) How much are the monthly production costs when no toaster ovens are being produced?

**Solution:** This is the value of \( y \) when \( x = 0 \), i.e., the \( y \)-intercept. It’s $3000.

(b) The company is currently producing 500 toaster ovens in a month. How much extra would it cost to produce one extra toaster oven in the month?

**Solution:** This question is asking how much \( y \) increases when \( x \) changes from 500 to 501. Since the graph is a line, it doesn’t actually matter that it changes from 500 to 501, just that it changes by 1. And to find out how much the \( y \)-value changes when the \( x \)-value changes by 1, we’re just looking for the slope of the line. Since it includes the points (0, 3000) and (1000, 9000), its slope is \( \frac{9000-3000}{1000-0} = 6 \). So the answer is that it would cost $6.

(c) Give an equation relating \( x \) and \( y \).

**Solution:** We’re looking for the equation of a line with slope 6 and \( y \)-intercept 3000:

\[
y = 6x + 3000
\]
9. You are going to use 400 feet of fencing to make a rectangular chicken coop.

(a) Find a function modeling the area $A(x)$ of the chicken coop in terms of its width $x$.

Solution: Let $x$ and $y$ be the width and length of the coop. The perimeter of the coop is $2x + 2y$, and we’re given that it’s 400. Rearranging $2x + 2y = 400$ gives $y = 200 - x$.

The area of the chicken coop is $xy$. Since $y = 200 - x$,

$$A(x) = x(200 - x) = -x^2 + 200x.$$

(b) What is area of the biggest coop you can make? What is its width?

Solution: We put $A(x)$ in standard form as

$$A(x) = -(x^2 - 200x) = -(x^2 - 200x + 10000) + 10000 = -(x - 100)^2 + 10000.$$

So, the maximum point on the parabola has $x$-value 100 and $y$-value 10000. That is, the biggest coop we can make has area 10000 sq. ft. and width 100 ft.