Math 123, Midterm 2, Version B
November 19, 2025

Name: ____________________

I pledge that I have neither given nor received unauthorized assistance during this examination.

Signature:

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.
- You may use a graphing or scientific calculator on this exam. You may not use a phone.
- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.
- There are 8 problems on 11 pages.

Question	Points	Score
1	15	
2	12	
3	10	
4	8	
5	8	
6	6	
7	8	
8	8	
Total:	75	

Good luck!

[15 points] 1. Solve the following equations and give **exact** answers. (Some of the exact answers will be in terms of of logarithmic or exponential functions.)

(Your work should be written as a list of equations, each following from the last one. Do not write down expressions with no indication of the relationship between them.)

(a) $(x+3)(x-1) = 17$

Solution: First expand and simplify:

$$x^2 + 3x - x - 3 = 17$$
$$x^2 + 2x - 3 = 17$$

Now subtract 17 from both sides:

$$x^2 + 2x - 20 = 0.$$

Now apply the quadratic formula:

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-20)}}{2} = \frac{-2 \pm \sqrt{84}}{2} = -1 \pm \sqrt{21}$$

The two answers numerically are $x \approx 3.58$ and $x \approx -5.58$.

(b) $9^x = 25$

Solution: Take logarithms of both sides (base-9 or natural are both fine):

$$x \ln 9 = \ln 25$$
$$x = \frac{\ln 25}{\ln 9}$$

(c) $10^{1-x} = 6^x$

Solution: Take logarithms of both sides:

$$(1-x)\ln 10 = x \ln 6$$

Expand the left-hand side:

$$\ln 10 - x \ln 10 = x \ln 6$$

Add $x \ln 10$ to both sides and factor:

$$\ln 10 = x(\ln 6 + \ln 10)$$

Divide both sides by $\ln 6 + \ln 10$:

$$x = \frac{\ln 10}{\ln 6 + \ln 10} = \frac{\ln 10}{\ln 60}.$$

(d) $\log_3(x+3) - \log_3(x-3) = 2$

Solution: Combine the logarithms as

$$\log_3\left(\frac{x+3}{x-3}\right) = 2$$

Now rewrite it as an exponential equation:

$$\frac{x+3}{x-3} = 3^2 = 9.$$

Multiply both sides by $x - 3$:

$$x + 3 = 9(x-3) = 9x - 27.$$

Subtract x and add 27 to both sides:

$$30 = 8x$$

Divide both sides by 8:

$$x = \frac{30}{8}.$$

(e) $(x+3)^2 = (x+1)^2 + (x+2)^2 + 1$

Solution: Expand as

$$x^2 + 6x + 9 = x^2 + 2x + 1 + x^2 + 4x + 4 + 1 = 2x^2 + 6x + 6.$$

Subtract $x^2 + 6x$ from both sides:

$$9 = x^2 + 6.$$

Subtract 6 from both sides:

$$3 = x^2.$$

So, $x = \pm\sqrt{3}$.

[12 points] 2. When transforming an expression into different forms, all of which are equal, your answer should be written as a chain of equalities. Do not write down expressions with no indication of the relationship between them.

(a) Combine the following expression into a single logarithm:

$$\ln(3x) - \ln(x^4+1) + 2\ln(x^2)$$

Solution:

$$\begin{aligned}\ln(3x) - \ln(x^4+1) + 2\ln(x^2) &= \ln(3x) - \ln(x^4+1) + \ln(x^4)\\ &= \ln\left(\frac{3x(x^4)}{x^4+1}\right) = \ln\left(\frac{3x^5}{x^4+1}\right)\end{aligned}$$

(b) Expand the following expression into an expression in the form $A\ln x + B\ln y + C\ln z$, where A, B, C are some values to be figured out:

$$\ln\left(\frac{x^4}{yz^5}\right)$$

Solution:

$$\begin{aligned}\ln\left(\frac{x^4}{yz^5}\right) &= \ln(x^4) - \ln(y) - \ln(z^5)\\ &= 4\ln x - \ln y - 5\ln z\end{aligned}$$

(c) Expand and simplify the following expression, and eliminate all negative exponents:

$$\left(x + x^{-1}\right)^2 - x$$

Solution:

$$\begin{aligned}\left(x + x^{-1}\right)^2 - x &= x^2 + 2xx^{-1} + x^{-2} - x \\ &= x^2 + 2 + \frac{1}{x^2} - x\end{aligned}$$

(d) Simplify as much as possible:

$$\log_a\left(\frac{1}{a^2}\right)$$

Solution:

$$\log_a\left(\frac{1}{a^2}\right) = -2$$

3. Let $f(x) = -9x^2 + 2x$.

[3 points] (a) Express f in standard form.

Solution:

$$\begin{aligned} f(x) &= -9(x^2 - \tfrac{2}{9}) + 1 \\ &= -9\big(x^2 - \tfrac{2}{9} + \tfrac{1}{81} - \tfrac{1}{81}\big) \\ &= -9\big(x^2 - \tfrac{2}{9} + \tfrac{1}{81}\big) + \tfrac{1}{9} \\ &= -9\big(x - \tfrac{1}{9}\big)^2 + \tfrac{1}{9} \end{aligned}$$

[1 point] (b) What is the extreme point of the graph (which is also known as the vertex of the graph)?

Solution: From the standard form, we see that it's (1/9, 1/9)

[2 points] (c) List all x-intercepts of f.

Solution: The x-intercepts are the solutions to $f(x) = 0$. We solve by factoring:

$$0 = -9x^2 + 2x = x(-9x + 2).$$

The right-hand side is equal to zero if $x = 0$ or if $-9 + 2x = 0$, which has solution $x = 2/9$. So the x-intercepts are 0 and 2/9.

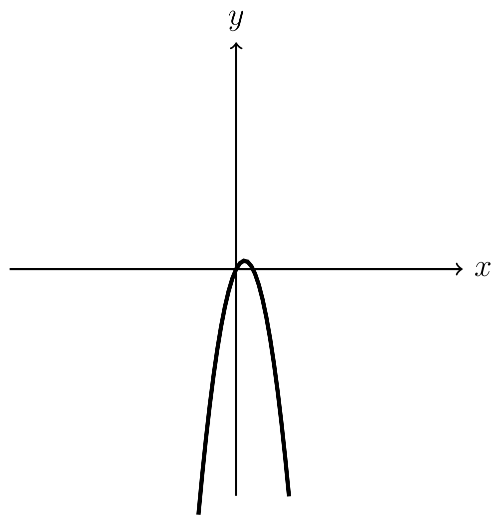
[2 points] (d) List all y-intercepts of f.

Solution: The y-intercept of $f(x)$ is found by plugging in $x = 0$:

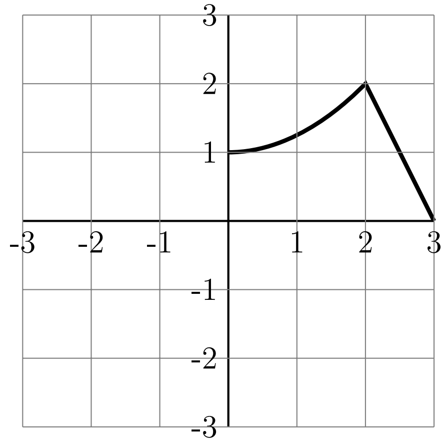
$$f(0) = 0$$

So the y-intercept is 0.

[2 points] (e) Sketch the graph $y = f(x)$ on these axes:


y
x

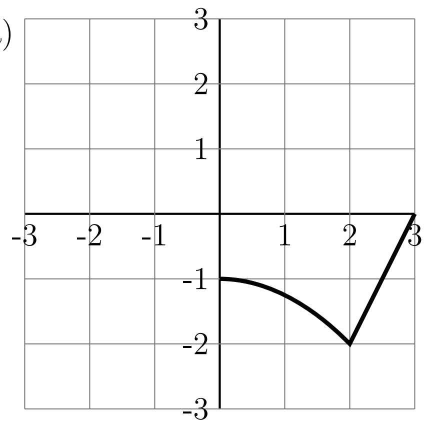
[8 points] 4. Here is the graph of the function $f(x)$ on the domain $[0, 3]$:



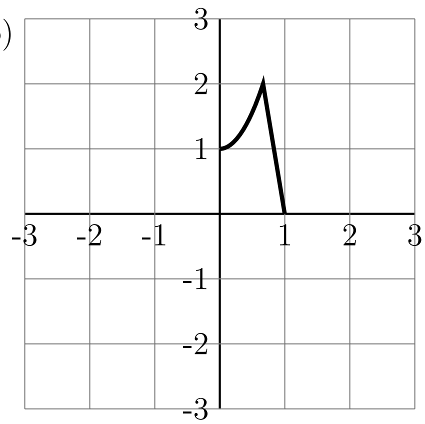
Sketch the graphs of the following functions on the axes below.

(a) $-f(x)$
(b) $f(3x)$
(c) $f(x+1) - 2$
(d) $f(-x) - 1$

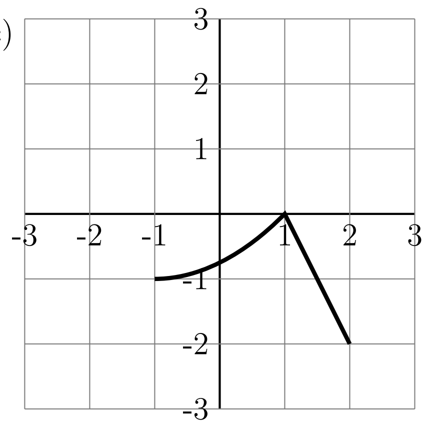
(a)



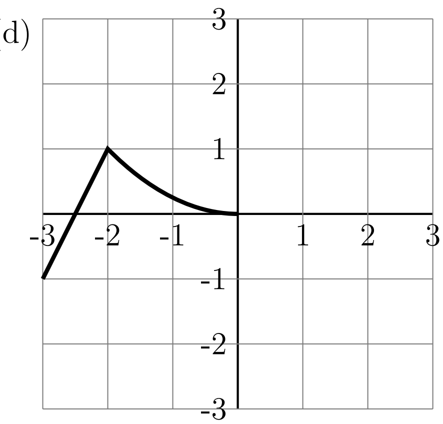
(b)



(c)



(d)



5. The isotope strontium-90 has a half-life of 28 years (i.e., its mass is halved every 28 years due to radioactive emission). You have a sample of 40 grams of strontium-90.

[3 points] (a) Give a function $m(t)$ that records the amount of strontium-90 remaining after t years.

Solution:

$$m(t) = (40)2^{-t/28}$$

[2 points] (b) How much strontium-90 remains after one year?

Solution:

$$m(1) = (40)2^{-1/28} \approx 39.022$$

[3 points] (c) How long will it take to reduce the strontium-90 to 4 grams?

Solution: We need to solve

$$m(t) = (40)2^{-t/28} = 4.$$

Divide both sides by 40 to get

$$2^{-t/28} = \frac{1}{10}.$$

Take logarithms to get

$$-\frac{t}{28} \log 2 = \log(1/10).$$

Now solve for t and get

$$t = -\frac{28 \log(1/10)}{\log 2} \approx 93.01 \text{ years}$$

[6 points] 6. A strain of bacteria is growing exponentially. Initially there are 8400 bacteria. After one hour, there are 10000.

(a) Give a function $A(t)$ that records the number of bacteria after t hours.

Solution: Our function will be of the form

$$A(t) = (8400)2^{t/a}$$

where a is the doubling time. To solve for a, we set $A(1) = 10000$ and solve

$$10000 = (8400)2^{1/a}.$$

Divide both sides by 8400:

$$\frac{10000}{8400} = 2^{1/a}$$

Take logarithms of both sides:

$$\log\left(\frac{10000}{8400}\right) = \frac{1}{a}\log 2.$$

So,

$$a = \frac{\log 2}{\log(10000/8400)} \approx 3.98,$$

And our equation is

$$A(t) = (8400)2^{t/3.98}.$$

(b) How long does it take the population to double?

Solution: From the previous part, it takes 3.98 years.

[8 points] 7. Michaela sells jam for \$10 per jar at the farmer's market. On average, she sells 50 jars per week. She estimates that for every dollar she decreases the price, she will sell an additional four jars.

(a) Give a function $R(x)$ modeling her revenue (total dollars taken in) in terms of the price x.

Solution: The total number of jars sold at price x is

$$50 + 4(10 - x) = 90 - 4x.$$

The total revenue at price x is then

$$R(x) = x(90 - 4x) = 90x - 4x^2.$$

(b) What price should she sell her jam for to maximize her revenue?

Solution: We want to find x maximizing $R(x)$. So we put $R(x)$ into standard form:

$$\begin{aligned} R(x) &= -4x^2 + 90x = -4(x^2 - 45/2) = -4(x^2 - 45/2 + 506.25 - 506.25) \\ &= -4(x^2 - 45/2 + 506.25) + 2025 = -4(x - 11.25)^2 + 2025 \end{aligned}$$

The function is maximized at $x = 11.25$.

[8 points] 8. Mark the following statements as **True** if they are always true (assuming the variables are positive numbers), and otherwise mark them as **False**. You do not need to show any justification.

(a) $\ln x + \ln y = \ln(x + y)$ ○ True √ **False**

(b) $(\ln x)(\ln y) = \ln(xy)$ ○ True √ **False**

(c) $(x + y)^2 = x^2 + y^2$ ○ True √ **False**

(d) $(xy)^2 = x^2y^2$ √ **True** ○ False

(e) $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$ ○ True √ **False**

(f) $\sqrt{xy} = \sqrt{x}\sqrt{y}$ √ **True** ○ False

(g) $\frac{x+y}{z} = \frac{x}{z} + \frac{y}{z}$ √ **True** ○ False

(h) $\frac{xy}{z} = \left(\frac{x}{z}\right)\left(\frac{y}{z}\right)$ ○ True √ **False**