Math 123, Midterm 1, Version A Name: ______________________________

October 15, 2025

I pledge that I have neither given nor received unauthorized assistance during this examination.

Signature:

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.
- You may use a graphing or scientific calculator on this exam. You may not use a phone.
- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.
- There are 9 problems on 9 pages.

Question	Points	Score
1	15	
2	6	
3	6	
4	6	
5	9	
6	4	
7	3	
8	6	
9	6	
Total:	61	

Good luck!

[15 points] 1. Find all solutions to the following equations and inequalities. For inequalities with a range of solutions, you can give your answer using any notation (e.g., you may say that the solution set is $[5, 6)$ or that it's given by $5 \le x < 6$). If no solutions exist, say so.

Your work should be written as a list of equations, each following from the last one.

(a) $x + 5 = 14 - 2x$

Solution: Add $2x$ to both sides of the equation and subtract 5 to get

$$3x = 9.$$

Now divide both sides by 3 to get $x = 3$.

(b) $3|x - 4| = 10$

Solution: We need to solve $3(x - 4) = 10$ and $3(x - 4) = -10$. For the first, we get

$$\begin{aligned} 3x - 12 &= 10 \\ 3x &= 22 \\ x &= \frac{22}{3}, \end{aligned}$$

and from the second we get

$$\begin{aligned} 3x - 12 &= -10 \\ 3x &= 2 \\ x &= \frac{2}{3}. \end{aligned}$$

And these are the two solutions.

(c) $3x^2 + x = 11$

Solution: Subtract 11 from both sides to rewrite the equation as

$$3x^2 + x - 11.$$

Now the quadratic formula gives

$$x = \frac{-1 \pm \sqrt{(-1)^2 - 4(3)(-11)}}{6}.$$

If you put this into a calculator, $x \approx -2.089$ and $x \approx 1.755$.

(d) $(x-3)(x-5) > 0$

Solution: The zeros of $(x-3)(x-5)$ occur at $x = 3$ and $x = 5$. So we just need to figure out the sign of $(x-3)(x-5)$ between these zeros, on the intervals $(-\infty, 3)$, $(3, 5)$, and $(5, \infty)$.

- On the interval $(-\infty, 3)$, both $(x-3)$ and $(x-5)$ are negative. So $(x-3)(x-5)$ is positive.
- On the interval $(3, 5)$, the expression $(x-3)$ is positive but $(x-5)$ is negative. So $(x-3)(x-5)$ is negative.
- On the interval $(5, \infty)$, both $(x-3)$ and $(x-5)$ are positive. So $(x-3)(x-5)$ is positive.

So, $(x-3)(x-5)$ is positive on $(-\infty, 3) \cup (5, \infty)$. Or you could say that it's positive if $x < 3$ or $x > 5$.

(e) $\dfrac{2x}{x+1} = \dfrac{2x-1}{x}$

Solution: Multiply both sides of the equation by $(x+1)(x)$. This gives

$$\frac{2x(x)(x+1)}{x+1} = \frac{(2x-1)(x)(x+1)}{x},$$

which simplifies to

$$2x^2 = (2x-1)(x+1).$$

Expand the right-hand side:

$$2x^2 = 2x^2 + 2x - x - 1.$$

Now subtract $2x^2$ from both sides to get

$$0 = 2x - x - 1 = x - 1.$$

And now add 1 to both sides to get

$$x = 1.$$

[6 points] 2. Simplify the following expressions as much as possible and eliminate any negative exponents. You can assume that the denominators of all fractions are nonzero and that anything whose root is being taken is positive.

When transforming an expression into different forms, all of which are equal, your answer should be written as a chain of equalities. Do not write down expressions with no indication of the relationship between them. (Note: if you can do the answer all in one step and just write down the final answer, that's fine. You don't need to bother repeating the original expression to write down that it's equal.)

(a) $\dfrac{2x^3y^6}{x^5y^2}$

Solution:

$$\frac{2y^4}{x^2}$$

(b) $(x^3 + x^{-2})^2 + x$

Solution:

$$\begin{aligned}(x^3 + x^{-2})^2 + x &= (x^3)^2 + 2x^3x^{-2} + (x^{-2})^2 + x \\ &= x^6 + 2x + x^{-4} + x \\ &= x^6 + 3x + \frac{1}{x^4}.\end{aligned}$$

[6 points] 3. Expand and simplify the following expressions.

When transforming an expression into different forms, all of which are equal, your answer should be written as a chain of equalities. Do not write down expressions with no indication of the relationship between them.

(a) $\left(\sqrt{x+2}+3\right)^2$

Solution:

$$\begin{aligned}\left(\sqrt{x+2}+3\right)^2 &= (\sqrt{x+2})^2 + 2\sqrt{x+2}(3) + 3^2\\ &= x+2+6\sqrt{x+2}+9 = x+11+6\sqrt{x+2}.\end{aligned}$$

(b) $(y+2)(y^2+y-1)$

Solution:

$$\begin{aligned}(y+2)(y^2+y-1) &= y^3+y^2-y+2y^2+2y-2\\ &= y^3+3y^2+y-2\end{aligned}$$

[6 points] 4. A rectangular field with area 2000 sq. ft. is surrounded by 200 feet of fencing. What are its dimensions?

Solution: Let x and y be the length and width of the field. We're given that

$$xy = 2000$$

and

$$2x + 2y = 200.$$

From the second equation, we can solve for y and get $y = 100 - x$. Plugging this into the first equation gives

$$x(100 - x) = 2000$$

which we can rearrange as

$$x^2 - 100x + 2000 = 0.$$

The quadratic formula gives

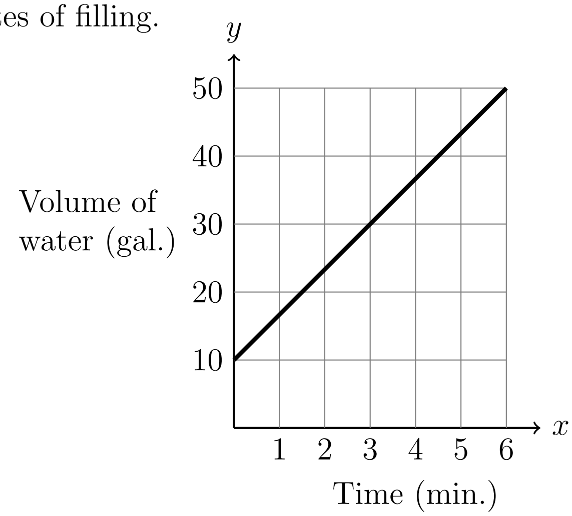
$$x = \frac{100 \pm \sqrt{(-100)^2 - 4(1)(2000)}}{2}$$

or

$$x \approx 72.36 \text{ or } x \approx 27.64.$$

Plugging back into y yields that either $x \approx 72.36$ and $y \approx 27.64$ or $x \approx 27.64$ and $y \approx 72.36$. So the dimensions are 72.36 ft. $\times$ 27.64 ft.

[9 points] 5. A swimming pool is being filled. The following graph shows the volume of water y in the pool after x minutes of filling.



(a) At what rate (measured in gallons per minute) is the pool being filled?

Solution: We need to find the slope of the line, which includes points $(3, 30)$ and $(0, 10)$. So the slope is

$$\frac{30-10}{3-0} = \frac{20}{3}.$$

(b) Give an equation expressing y in terms of x.

Solution: We already have the slope. The y-intercept is 10, as we can see on the graph. So the line is the solutions to the equation

$$y = \frac{20}{3}x + 10.$$

(c) At what time x will the pool have 100 gallons of water?

Solution: We want to solve the equation

$$100 = \frac{20}{3}x + 10.$$

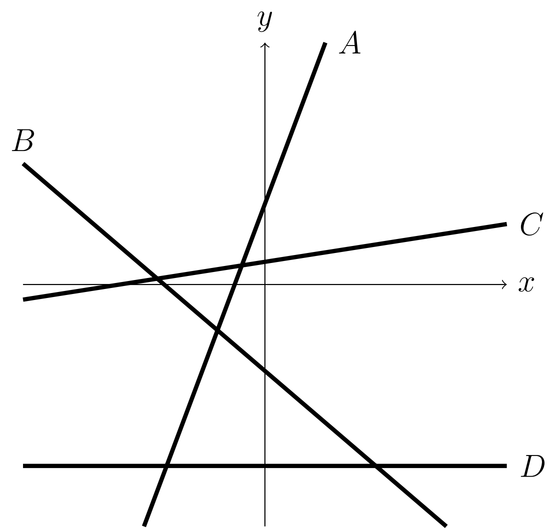
Subtracting 10 from both sides gives

$$\frac{20}{3}x = 90.$$

Multiplying both sides by $3/20$ gives

$$x = 90\left(\tfrac{3}{20}\right) = \frac{270}{20} = \frac{27}{2} = 13.5.$$

[4 points] 6. Here is a graph showing lines A, B, C, and D:



Put lines A, B, C, and D in order according to their slopes, from smallest to largest. (Note that some of the slopes may be negative.) Your answer should be a list A, B, C, D, in some order.

Solution: B, D, C, A

[3 points] 7. What is the domain of the function $f(x) = \dfrac{x+1}{x^2+3x}$?

Solution: Factor to rewrite the fraction as

$$f(x) = \frac{x+1}{x(x+3)}.$$

The denominator is zero when $x = 0$ or $x = -3$. So the domain is all real numbers except for 0 and -3.

[6 points] 8. Consider the piecewise function

$$f(x) = \begin{cases} -2 & \text{for } x < -3, \\ 2 & \text{for } -3 \le x \le 0, \\ x & \text{for } x > 0. \end{cases}$$

(a) What is $f(-4)$?

Solution: $f(-4) = -2$

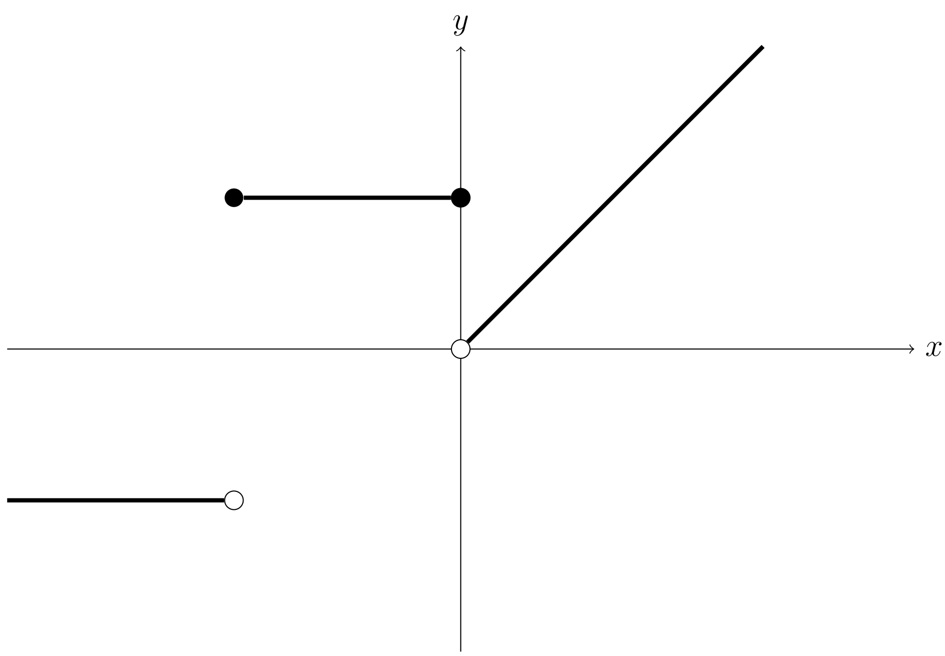
(b) What is $f(-3)$?

Solution: $f(-3) = 2$

(c) What is $f(2)$?

Solution: f(2) = 2

(d) Sketch a graph of $y = f(x)$ on the axes below.



[6 points] 9. New York and Washington are 250 miles apart. A train goes from New York to Washington and back. The train goes 10% faster when returning to New York as when going to Washington. The total roundtrip time is 9 hours. How fast does the train go on the way to Washington?

Solution: Let s be the speed of the train on the way to Washington. Then the speed on the way back to New York is $1.1s$. Since rate $\cdot$ time $=$ distance, the time from New York to Washington is $\frac{250}{s}$. The time from Washington to New York is $\frac{250}{1.1s}$. So,

$$\frac{250}{s} + \frac{250}{1.1s} = 9.$$

We multiply both sides of the equation by s to get

$$250 + \frac{250}{1.1} = 9s,$$

or

$$477.27 = 9s.$$

Dividing both sides by 9 gives $s = 53.03$ mph.