Problem 1

1. Give a short derivation of the Cauchy-Riemann equations.
2. Assume an analytic function f with
$$f(x + iy) = \phi + i\psi$$
and define $u = \nabla\phi$.
   (a) Show that $\Delta\phi = \Delta\psi = 0$.
   (b) Show that $\nabla\phi \cdot \nabla\psi = 0$.
   (c) How can you express u in terms of ψ?

Problem 2

Find the Fourier series of $f(x) = x^2$ for $-\pi < x \leq \pi$. Using this series, derive the following formula

$$\frac{\pi^2}{12} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} \ldots$$

Problem 3

Compute the following integral:

$$J = \int_0^{2\pi} \frac{d\theta}{\frac{5}{4} + \sin\theta}$$

Problem 4

Compute the following Fourier transform:

$$\frac{1}{\sqrt{2\pi}} \int \frac{1}{a^2 + x^2} \mathrm{e}^{-ikx}\, dx$$

Problem 5

Solve the following initial value problem using a Laplace transform. Use residues in order to calculate the inverse Laplace transform.

$$y'' + 4y' + 5y = 0, \qquad y(0) = 0,\ y'(0) = 2$$