Problem 1 (10 points)

1. Solve the transport equation using Fourier transform:

$$u_t + vu_x = 0, \qquad u(t=0,x) = f(x)\,.$$

2. Compute the convolution of the two Gaussians f and g analytically:

$$f(x) = \mathrm{e}^{-3x^2}, \qquad g(x) = \mathrm{e}^{-4x^2}\,.$$

Then write a MATLAB code that, using FFT, computes the convolution numerically and compare the results.

Problem 2 (10 points)

1. Consider $f(x) = \exp(2x)$ in $[0,1]$ and compute the Fourier cosine series.
2. Consider f given by

$$f(x) = \begin{cases} 1 & 0 \le x \le \pi/2 \\ 2 & \pi/2 < x \le \pi \end{cases}$$

and compute the Fourier sine expansion. Using matlab, plot the function itself and also its 10th and 30th partial sum.

Problem 3 (10 points)

1. Show that

$$\mathcal{L}^{-1}\left(\frac{\mathrm{e}^{-2\alpha\sqrt{s}}}{s}\right) = \mathrm{erfc}\left(\frac{\alpha}{\sqrt{t}}\right)\,.$$

Here, $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x) = 1 - \frac{2}{\sqrt{\pi}}\int_0^x \mathrm{e}^{-u^2}\,du$ is the *complementary error function.*

2. Solve the following partial differential equation given on $x > 0$ and $t > 0$ using Laplace transform:

$$u_t = \kappa u_{xx}\,,$$

and conditions $u(0,t) = u_0 = \text{const.}$ for $t > 0$ and $u(x,0) = 0$ for $x > 0$.