Problem 1 (10 points)

Consider energy conservation and conservation of angular momentum for the central force problem given by

$$E = \frac{\mu}{2}\left(\dot{r}^2 + r^2\dot{\phi}^2\right) + V(r)$$
$$L = \mu r^2 \dot{\phi}$$

1. Derive a first-order system of ODEs for the vector $(r, \dot{r}, \phi, \dot{\phi})$.
2. Use the Kepler-problem $V(r) = \alpha/r$ and the Kepler-problem with relativistic correction $V(r) = \alpha/r + \beta/r^2$ as an example and show using a numerical simulation that the motion leads to an ellipse and precession of elliptical orbits. Hand in your code and a printout of the graph.

Problem 2 (10 points)

Consider the linear inhomogeneous differential equation

$$\ddot{x} + x = -\cos(t)$$

and find the general solution using the method of variation of the constants.

Problem 3 (10 points)

Assume in the following that the matrix A can be diagonalized. This means that there exist a matrix M and a diagonal matrix D such that $A = MDM^{-1}$. D contains the eigenvalues of A.

1. Show that $\exp(A) = M\exp(D)M^{-1}$.
2. Assume now that all eigenvalues are positive. Show that

$$\int_0^\infty \exp(-sA)\, ds = A^{-1}$$