Due: Wed Sep 21 2016 Math Methods Homework 1

Problem 1 (10 points)

Find the analytical solution to the initial value problem given by

$$\dot{N} = rN\left(1 - \frac{N}{K}\right), \qquad N(0) = N_0 > 0$$

Assume that $r > 0$ and $K > 0$. Sketch a graph of the solution.

Problem 2 (10 points)

Solve the differential equation in problem 1 by using

1. the forward Euler-scheme,
2. the midpoint method,
3. the 4-th order Runge-Kutta.

You are allowed to modify the code from the example in class. For each method, plot a graph of the solution and the difference of the numerical solution and the analytical solution found in problem 1. How can one obtain the order of the numerical scheme for these solutions?

Problem 3 (10 points)

Consider the nonlinear initial value problem given by

$$\ddot{y} + \epsilon\dot{y}^3 + y = 0, \qquad y(0) = 0,\ \dot{y}(0) = 1$$

for small ϵ. Using the method of multiple scales, develop an approximation valid for long times up to the first nontrivial order in ϵ. Use Matlab to solve the problem numerically and plot the difference between the numerical solution and the approximation for the first 10 periods. Attach the graph and the Matlab script to your solution.