Problem 1

A stock process starts at time 0 at 100 and then can go 30 up or 10 down for each time step. We consider two time steps, the risk-free interest rate is 10%. Consider a European call option with strike price of \$100.

1. Sketch the Stock process.
2. Find the option value at all nodes of the tree.
3. Assume that the stock first goes down and then goes up. Compute the necessary holdings (ϕ, ψ) of stock and bond at each time step to hedge the above option.

Problem 2

Assume that W_t is a $\mathbb{P}$-Brownian motion.

1. Compute
$$\mathbb{E}_{\mathbb{P}}\left(\mathrm{e}^{3W_t+t}\right) .$$
2. Show, using Ito's lemma, that
$$\mathbb{E}_{\mathbb{P}}(W_t^4) = 3t^2 .$$
3. Compute
$$\int_{\tilde{t}}^{t} \int_{\tilde{t}}^{s} dW_u \, dW_s .$$

Problem 3

Consider a $\mathbb{P}$-Brownian motion W_t and a stock model given by

$$S_t = \sigma(W_t^2 - 2W_t t + t^2 - t) + S_0$$

1. Use Ito's lemma to check whether S_t is a $\mathbb{P}$-martingale.
2. Use the Girsanov theorem to construct a measure $\mathbb{Q}$ and a $\mathbb{Q}$-Brownian motion $\tilde{W}_t$ such that S_t is a $\mathbb{Q}$-martingale. Write S_t in terms of $\tilde{W}_t$.
3. Consider a claim X that pays \$100 if S_t is larger that $2S_0$ at maturity T. Moreover, assume that there are no interest rates. Find the price of X.