

**Problem 1 (10 points)**

Show that the Lorentz transformations form a group by proving ( $\beta \equiv v/c$ )

$$L_{\beta_1} \circ L_{\beta_2} = L_{\beta}, \quad \beta = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2}.$$

**Problem 2 (10 points)**

A particle of rest mass  $m$  and initial velocity  $v_0$  along the  $x$ -axis is subject after  $t = 0$  to a constant force  $F$  acting in the  $y$ -direction.

1. Find the magnitude of the velocity of the mass  $m$  at any time  $t$ . Show that  $|\mathbf{v}| \rightarrow c$  for  $t \rightarrow \infty$ .
2. Assuming that the particle started at  $t = 0$  at the origin  $(0, 0)$ , find the  $x$  and the  $y$  coordinate of the particle at any time  $t$ .

**Problem 3 (10 points)**

Assume a Hamilton function  $H$  of a particle of mass  $m$  given by

$$H(q, p) = c\sqrt{m^2c^2 + p^2} - \lambda q$$

1. Find Hamilton's equations of motion and solve them for the initial conditions  $q(0) = \dot{q}(0) = 0$ .
2. Find  $H$  for the non-relativistic limit  $p/(mc) \rightarrow 0$ .