Problem 1 (10 points)
Consider a point mass m under the influence of a potential V given in cylindrical coordinates (r, ϕ, z) as

$$V = \frac{\gamma}{\sqrt{r^2 + z^2}} - Fz$$

with positive constants γ and F.

1. Find the Lagrangian in *parabolic* coordinates (ξ, η, ϕ). Hint: Parabolic coordinates are related to cylindrical coordinates (r, ϕ, z) through the relations

$$r = \sqrt{\xi\eta}, \qquad z = \frac{1}{2}(\xi - \eta), \qquad \phi = \phi\,.$$

2. Find the Hamiltonian in parabolic coordinates.
3. Show that ϕ is cyclic. Call $p_\phi =: \alpha_2$.
4. Set up the Hamilton-Jacobi equation to find the generating function S in the form

$$S = S(\xi, \eta, \phi, E, \alpha_2, \alpha_3, t)$$

5. Find S in terms of quadratures (you do not need to find closed-form solutions of the occuring integrals).

Problem 2 (10 points)
Show that for an n-dim. Hamiltonian system, the invariance of the Poisson brackets implies that the transform under consideration is canonical.
Hint: Set $x = (q_n, p_n)$ and $y = (Q_n, P_n)$ and show that from the invariance of the Poisson brackets we can see that $J = MJM^T$ if M is the Jacobian of the transformation.

Problem 3 (10 points)
The Hamiltonian for a systems of two particles is given by

$$H = \frac{1}{2m}(p_1^2 + q_2^2) + \frac{k}{2}(p_2^2 + q_1^2)$$

Solve for the motion of the system using the Hamilton-Jacobi method. Assume that at $t = 0$, $p_1 = 0$, $q_1 = A$, $p_2 = B$, $q_2 = 0$.