**Problem 1 (10 points)**
A uniform ladder of mass $M$ and length $2L$ is leaning against a frictionless vertical wall with its feet on a frictionless horizontal floor. Initially the stationary ladder is released at an angle $\theta_0 = 60^{\rm o}$ to the floor. Assume that the gravitation field $g$ acts vertically downward.

1. (3 points) Show that the moment of inertia of the ladder about its midpoint is $I = \frac{1}{3}ML^2$.
2. (2 points) Derive the Lagrangian of the system.
3. (1 point) Derive the equations of motions using the Lagrangian.
4. (4 points) Find the angle $\theta$ at which the ladder loses contact with the vertical wall.

**Problem 2 (10 points)**
Find the principal moments of inertia of an ellipsoid.

**Problem 3 (10 points)**
Consider the symmetric heavy spinning top with the Lagrangian

$$L = \frac{1}{2}I_1(\dot{\phi}^2 \sin^2\theta + \dot{\theta}^2) + \frac{1}{2}I_3(\dot{\psi} + \cos\theta\dot{\phi})^2 - Mgl\cos\theta$$

1. Find three conserved quantities and use them in order to rewrite the equation of motion for $\theta$ in terms of $u = \cos\theta(t)$ as
$$\frac{1}{2}\dot{u}^2 + V_{\rm eff}(u) = 0.$$
2. Sketch the effective potential for parameters of your choice and discuss the motion qualitatively.
3. A so-called "sleeping top" with $\theta = 0$ is stable if it is spinning faster than a critical angular velocity $\omega_c$. Find $\omega_c$.