Problem 1 (10 points)
Consider the motion of a mass m in a potential given by $V(\vec{x}) = \alpha/\vec{x}^2$.

1. Show that the transformation given by

$$x' = (1+\epsilon)x, \qquad t' = (1+\epsilon)^2 t$$

   is a symmetry of the Lagrangian.

2. Using Noether's Theorem, construct the corresponding conserved quantity.

3. Using the Euler-Lagrange equations, show that the above constructed quantity is indeed a constant of motion.

Problem 2 (10 points)
Consider the linear model of a molecule with three atoms. The outer atoms are of mass m and the atom in the molecule's center is of mass M. The outer atoms are connected to the center atom through springs of a constant k.

1. Find the Lagrange function of the system. Use as coordinates the deviations of the atoms from their equilibrium position.

2. Find the eigenfrequencies of the system and corresponding eigenvectors.

3. Write down the general solution of the equation of motion of the system.

Problem 3 (10 points)
A linear chain consists of $2N$ equally spaced identical particles with mass m and separation a, coupled by springs with alternating constants k_1 and k_2. Use the periodicity of the system to find the dispersion relation and sketch the resulting function $\omega(q)$. Hint: Assume

$$\eta_{2j} = \alpha \mathrm{e}^{i[2jaq-\omega t]}, \qquad \eta_{2j+1} = \beta \mathrm{e}^{i[(2j+1)aq-\omega t])}$$