Problem 1 (10 points)

A bead of mass slides without friction on a wire whose shape is

$$z(r) = a\left(\frac{r}{a}\right)^4$$

The wire rotates about the z axis with constant velocity ω. Gravity causes acceleration g along the $-z$ axis.

1. Using cylindrical coordinates (r, ϕ, z), write the Lagrangian of the system.
2. Find the equation of motion for the bead in terms of the coordinate r.
3. For the given parameters a, ω, g, show that a circular orbit is possible and find its radius r_0.
4. Find the frequency of small oscillations about this orbit using $r = r_0 + A\sin(\Omega t)$.

Problem 2 (10 points)

Consider again the Lagrangian L given by

$$L = T - U = \frac{1}{2}mv^2 - q\left(\phi - \vec{v}\cdot\vec{A}\right)$$

1. Repeat the calculations in class for the y-component of the Lorentz-force and show that the above defined Lagrangian leads to the correct equation of motion.
2. Consider a gauge transform using a scalar field $\chi = \chi(\vec{r}, t)$:

   $$\vec{A}' = \vec{A} + \nabla\chi, \qquad \phi' = \phi - \frac{\partial\chi}{\partial t}$$

   and show that the transformed Lagrangian L' is given by

   $$L' = L + q\frac{d}{dt}\chi$$

Problem 3 (10 points)

A film of soap is streched between two coaxial circular rings of equal radius R. The distance between the rings is d. Ignore gravity. Find the shape of the soap film. Hint: Assume that the shape will correspond to a minimal surface and that the system has cylindrical symmetry.