Analytical Dynamics
Fall 2020
Exam I

10/26/2020

1/ A particle moves under the influence of a cental force whose potential is $V(r) = -V_0\, e^{-\lambda^2 r^2}$ $(V_0 > 0)$.

(a) (10 points) Write down the Lagrangian in polar coordinates and find the effective potential. Sketch the effective potential.

(b) (10 points) For a given angular momentum ℓ, find the radius of the stable orbit (an implicit equation is ok.)

(c) (10 points) It turns out that, if ℓ is too large, then no circular orbit exists. What is the largest value of ℓ for which a circular orbit does in fact exist?

2/ A particle of mass 1 follows the motion prescribed by the Lagrangian $\mathcal{L} = \frac{1}{2}\left((\dot{x} + ax)^2 + (\dot{y} + ay)^2\right)$ with $a > 0$. At time $t \to -\infty$, the particle is at $(0,0)$ and at time $t = 0$ it is at $(2,3)$.

(a) (10 points) Derive the corresponding Euler-Lagrange equations.

(b) (10 points) Solve the equations with the boundary conditions specified above.

(c) (10 points) Compute the value of the action S, given by

$$S = \int_{-\infty}^{0} \mathcal{L}\, dt.$$

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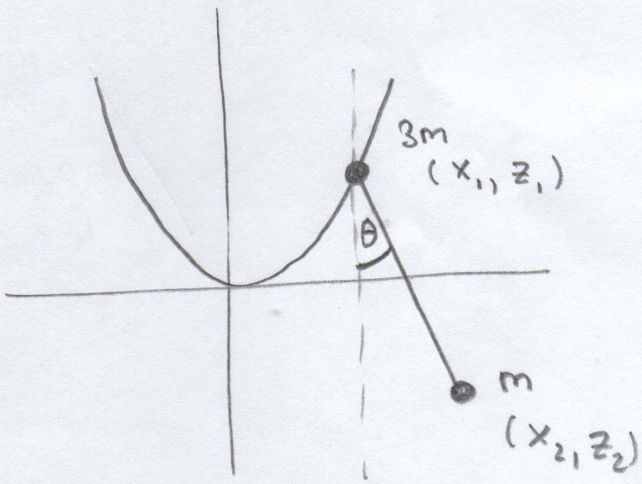
3/ In a uniform gravitational field, consider a point mass m with coordinates (x_2, z_2) that is attached to a point mass $3m$ with coordinates (x_1, z_1) through a massless string of length l. The mass $3m$ moves on a parabola of the form $z_1 = \frac{1}{2l} x_1^2$ (see figure). The goal is to find the eigenfrequencies for small oscillations $(\theta \ll 1)$. Choose $q_1 = x_1$ and $q_2 = l\theta$ as generalized coordinates.



(a) (10 points) Express (x_1, z_1) and (x_2, z_2) in terms of q_1 and q_2.

(b) (15 points) Find the Lagrangian $\mathcal{L}(q_1, \dot{q}_1, q_2, \dot{q}_2)$. Remember that the oscillations are small and neglect terms of cubic or higher order.

(c) (15 points) Find the eigenfrequencies of the system.