**Discussion November 19, 2024** **Review Sheet Exam II**

1. Consider the permutation $\alpha = (1,3,4,8)(2,5,7)$. Compute $\alpha^2$ and $\alpha^{2019}$.
2. Let $G = \mathbb{Z}_6$ and $H = \{0,3\}$. Find all cosets of $H$ in $G$ and determine $[G : H]$.
3. Find all integers $x$ such that $x^{86} = 6 \mod 29$. Hint: Show first that $x^{28} = 1 \mod 29$
4. Prove that the image of the neutral element under a homomorphism is the neutral element.
5. Let $G$ be a group and $a \in G$. Define $\phi(x) = axa^{-1}$. Show that $\phi$ is an automorphism.
6. Prove that $\mathbb{Z}$ under addition is not isomorphic to $\mathbb{Q}$ under addition.
7. Find all the left cosets of $\{1, 11\}$ in $U(30)$.
8. Prove that any group of order 55 must have exactly one subgroup of order 5 or exactly 11 subgroups of order 5.
9. If $N$ is a normal subroup of $G$ and $|G/N| = m$, show that $x^m \in N$ for all $x \in G$.
10. Suppose that $\phi : \mathbb{Z}_{50} \to \mathbb{Z}_{15}$ is a group homomorphism with $\phi(7) = 6$. Determine $\phi^{-1}(12)$, the est of all elements that map to 12.