Discussion October 1, 2024 **Review Sheet Exam I**

1. Prove or disprove the following statement: Assume G is a group. If for all $a \in G$ we know that $a^2 = e$ (where e is the neutral element), then G is Abelian. Is the converse statement true?
2. For $a, b \in \mathbb{R}$, define the operation $\bigotimes$ as $a \bigotimes b = ab - 4$. Is this operation associative?
3. Consider $H = \{0, 2, 4\} \subset \mathbb{Z}_6$. Is H a subgroup of $\mathbb{Z}_6$?.
4. Prove that the neutral element of a group is unique.
5. Write down the elements of $U(22)$. What is $7 \cdot 21$ in $U(22)$? What is the inverse element of 3?
6. Find all solutions to $z^5 = 32$ in $\mathbb{C}$.
7. List all the elements of $\mathbb{Z}_{12}$ and, for each element, find its order.
8. If a, b, c are group elements and $|a| = 6$, $|b| = 7$, express $(a^4c^{-2}b^4)^{-1}$ without negative exponents.
9. Show that $U(14)$ is cyclic.
10. Prove by induction that, for all positive integers n, $n^3 + (n+1)^3 + (n+2)^3$ is a multiple of 9.