Due: Oct 31 2024 **MTH 339 Algebra Homework 5**

1. (10 pts) Let $H = \{0, \pm 3, \pm 6, \pm 9, ...\}$ Find all the left cosets of H in $\mathbb{Z}$.
2. (10 pts) Compute $5^{15} \mod 7$ and $7^{13} \mod 11$.
3. (10 pts) Let G be a group of order 60. What are the possible orders for the subgroups of G?
4. (10 pts) Let G be a group with $|G| = pq$ where p and q are prime. Prove that every proper subgroup is cyclic.
5. (10 pts) Suppose G is an Abelian group with an odd number of elements. Show that the product of all elements is the identity.
6. (10 pts) Prove that $\mathbb{Z} \bigoplus \mathbb{Z}$ is not cyclic.
7. (10 pts) Show that the mapping $\phi : U(st) \to U(s) \bigoplus U(t)$ given by $\phi(x) = (x \mod s, x \mod t)$ is indeed an isomorphism.
8. (10 pts) Let p be a prime. Prove that $\mathbb{Z}_p \bigoplus \mathbb{Z}_p$ has exactly $p+1$ subgroups of order p.
9. (10 pts) Express $U(165)$ as an external direct product of U-groups in four different ways.
10. (10 pts) What is the smallest positive integer k such that $x^k = e$ for all $x \in U(7 \cdot 17)$?