Due: Oct 22 2024 **MTH 339 Algebra Homework 4**

1. (10 pts) Prove that a group of order 3 must be cyclic.

2. (20 pts) Consider the following permutations:

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix} \qquad \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$$

   Compute α^{-1}, $\beta\alpha$, and $\alpha\beta$.

3. (20 pts) Consider the following permutations:

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix} \qquad \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$$

   Compute α, β, and $\alpha\beta$ as products of cycles.

4. (10 pts) Let G be a group of permutations on a set X. Let $a \in X$, and define $\operatorname{stab}(a) = \{\alpha \in G : \alpha(a) = a\}$ (stabilizer of a in G). Prove that $\operatorname{stab}(a)$ is a subgroup of G.

5. (10 pts) Show that $\phi(x) = \sqrt{x}$ is an automorphism of $\mathbb{R}^+$ under multiplication.

6. (10 pts) Show that $U(8)$ is not isomorphic to $U(10)$.

7. (10 pts) Show that $U(8)$ is isomorphic to $U(12)$.

8. (10 pts) Let G be a group under multiplication, $\bar{G}$ a group under addition and ϕ an isomorphism $G \to \bar{G}$. If $\phi(a) = \bar{a}$ and $\phi(b) = \bar{b}$, express $\phi(a^3b^{-2})$ in terms of $\bar{a}$ and $\bar{b}$.