Due: Sep 26 2024 **MTH 339 Algebra Homework 3**

1. (10 pts) Consider the set G_n of complex roots of unity (solutions to $z^n - 1 = 0$) and show that G_n is a group under multiplication.
2. (10 pts) Show that G_4 is a subgroup of G_8. Sketch the elements of both groups on the complex plane.
3. (10 pts) Prove that the center $Z(G)$ of a group G is a subgroup of G.
4. (10 pts) Prove that the centralizer $C(a)$ is a subgroup.
5. (10 pts) Prove that $Z(G) = \bigcap_{a \in G} C(a)$.
6. (10 pts) Consider $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in SL(2, \mathbb{R})$. Find $|A|$.
7. (10 pts) Consider $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in SL(2, \mathbb{Z}_p)$, p prime. Find $|A|$.
8. (10 pts) Let $G = GL(2, \mathbb{R})$ and
$$H = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \right\}$$
where a, b are nonzero integers. Prove or disprove that H is a subgroup of G.
9. (10 pts) Let $G = GL(2, \mathbb{R})$. Find $C\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right)$.
10. (10 pts) Let $G = GL(2, \mathbb{R})$. Find $Z(G)$.