Problem 1

Biased wheel of fortune: Assume that the probability density (for an angle θ) is given by $p(\theta) = \lambda \theta$. I pay \$1000 times the angle, X is the corresponding pay-off is X.

- 1. Find the constant λ .
- 2. Find the expectation value and the standard deviation of X.

Problem 2

A stock process starts at time 0 at 100 and then can go 40 up or 20 down for each time step. We consider two time steps, the risk-free interest rate is zero. Consider a European call option with strike price of \$150.

- 1. Sketch the Stock process.
- 2. Find the option value at all nodes of the tree.
- 3. Assume that the stock first goes up. Find the appropriate hedging strategy for this case and compute your stock and bond holdings. Explain in detail how you are hedged for the next time-tick, independent of what is going to happen next.

Problem 3

A stock price is currently \$25. It is known that at the end of 2 months it will be either \$23 or \$27. The risk-free interest rate is 10% per annum with continuous compounding. Suppose S_T is the stock price at the end of 2 months. What is the value of a derivative that pays off S_T^2 at this time?

Problem 4

What is the price of a European call option on a non-dividend-paying stock when the stock price is \$52, the strike price is \$50, the risk-free interest rate is 12% per annum, the volatility is 30% per annum, and the time to maturity is 3 months? Use the Black-Scholes formula.

Problem 5

Consider a stock process S that starts at time 0 at 100 and then can go up and down 20 for each time step. We consider three time steps, the risk-free interest rate is zero.

- 1. Find a measure \mathbb{Q} such that S is a martingale with respect to \mathbb{Q} .
- 2. Consider X, the claim, to be the European call at a strike price 100 at time i = 3. Find the process the process Y defined as $Y_i = \mathbb{E}_{\mathbb{Q}}(X|\mathcal{F}_i)$.
- 3. Show explicitly that

 $\mathbb{E}_{\mathbb{O}}(Y_3|\mathcal{F}_1) = Y_1$

for all possible filtrations \mathcal{F}_1 .