

**Problem 1 (10 points)**

1. Show that if  $B_t$  is a zero-volatility process and  $X_t$  is any stochastic process, then

$$d(B_t X_t) = B_t dX_t + X_t dB_t$$

2. Use Ito's formula and the rule above to check whether the following process

$$Y_t = W_t^3 - 3tW_t$$

is a martingale.

**Problem 2 (10 points)**

What is the solution of

$$dX_t = X_t(\sigma dW_t + \mu \sin(t)dt), \quad X_0 = a > 0$$

where  $\sigma$  and  $\mu$  are assumed to be constants?

**Problem 3 (10 points)**

Consider the stochastic differential equation (SDE)

$$dX_t = \sigma dW_t + \mu dt, \quad X_0 = a > 0$$

where  $\sigma$  and  $\mu$  are constants.

1. Write down the solution  $X_t$  of this SDE.
2. Find mean and expectation value of  $X_t$ .
3. Write down the distribution function  $p(x, t)$  of  $X_t$ .