Due: Mon Oct 7 2019 **MTH 416 Homework 3**

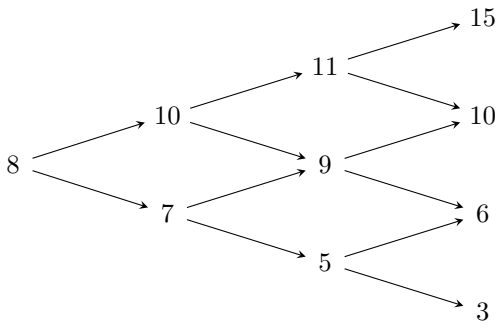
Problem 1 (10 points)

A stock process starts at time 0 at 100 and then can go 30 up or 10 down for each time step. We consider two time steps, the risk-free interest rate is 10%. Consider a European call option with strike price of \$100.

1. Sketch the Stock process.
2. Find the option value at all nodes of the tree.
3. Assume that the stock first goes down and then goes up. Compute the necessary holdings (ϕ, ψ) of stock and bond at each time step to hedge the above option.

Problem 2 (10 points)

Consider the following stock process S_i at times $i = 0, 1, 2, 3$.



1. Find a measure $\mathbb{Q}$ such that S becomes a $\mathbb{Q}$-martingale.
2. Find $\mathbb{E}_{\mathbb{Q}}(X_2|\mathcal{F}_1)$ and $\mathbb{E}_{\mathbb{Q}}(X_3|\mathcal{F}_1)$ for the filtration $\mathcal{F}_1$ that corresponds to the node where $S_1 = 10$ for the following processes X:
   (a) $X_i = 2S_i$
   (b) $X_i = S_i^2 - S_i$

Problem 3 (10 points)

Consider a stock process S on a tree. S starts at time $i = 0$ at $S_0 = 77$ and it can go up or down 30% every year. The interest rate is 2%.

1. Sketch the stock process for two years.
2. Consider a European call option at a strike price of $K = 88$ and with an expiration date of two years. Find the value of the option at all nodes of the tree.
3. Consider now a stock process following the Black-Scholes model with a volatility of 30%. If all other parameters are the same as before in the discrete model, what will be the price of the option according to the Black-Scholes formula?