Problem 1 (10 points)

Consider the option pricing formula for the binomial branch model given in class

$$V = \frac{f_u - f_d}{s_u - s_d}\, s_{now} + \mathrm{e}^{-r\delta t}\left(\frac{f_d s_u - f_u s_d}{s_u - s_d}\right)$$

1. Show that this formula is indeed algebraically equivalent to

$$V = \mathrm{e}^{-r\delta t}\left(q f_u + (1-q) f_d\right), \qquad q = \frac{\mathrm{e}^{r\delta t} s_{now} - s_d}{s_u - s_d}\,.$$

2. Use the formula to price a *forward* contract, i.e. a claim $X = S_T - K$.

Problem 2 (10 points)

Consider the example of a stock that starts at \$100 and can go up or down \$20 per year and a bond of \$100 which will be worth \$110 after one year.

1. Find the price of a European call with $K = \$100$ and a maturity of one year.
2. Show that, with the correctly computed option price, the example portfolio in the lecture notes will not be an arbitrage opportunity. Note: To set up the portfolio, buy one option and 2/5 of the bond and short sufficient stock to obtain the portfolio at no cost. Then compute the value of the portfolio after one year.

Problem 3 (10 points)

A stock process starts at time 0 at 100 and then can go 30 up or 10 down for each time step. We consider three time steps, the risk-free interest rate is zero. Consider a European call option with strike price of \$100.

1. Sketch the Stock process.
2. Find the option value at all nodes of the tree.
3. Assume that the stock first goes down and then goes up. Compute the necessary holdings (ϕ, ψ) of stock and bond at each time step to hedge the above option.