Problem 1 (10 points)

An investor agrees to sell insurance for a portfolio of 100 identical mortgages against defaults. Assume independence and that the random variable X counting the number of defaults has a binomial distribution with $p = 0.1$. The investor agrees to pay a flat fee of \$10M if X is between 10 and 15, including these values.

1. (1 point) Compute $P(X = 8)$, μ_X, and σ_X.
2. (2 point) Find the exact $P(10 \leq X \leq 15)$ using the binomial distribution.
3. (3 points) Find the approximate $P(10 \leq X \leq 15)$ using the normal approximation of the binomial distribution.
4. (1 point) What would be the 'fair' price for the insurance contract?
5. (3 points) How does this price change if $p = 0.2$?

Problem 2 (10 points)

A continuously distributed random variable X has the probability density given by $p(x) = a \exp(-\lambda x)$ for $x \in [0, \infty)$ and zero elsewhere. Assume $\lambda > 0$.

1. Find the constant a.
2. Find the expectation value of X.
3. Find $\mathbb{E}((X-1)^+)$.

Problem 3 (10 points)

Show that the following formula holds for a random variable X with a Gaussian distribution (mean μ and variance σ^2), and for a real number θ:

$$\mathbb{E}\left(\mathrm{e}^{\theta X}\right) = \exp\left(\theta\mu + \frac{1}{2}\theta^2\sigma^2\right)$$

Hint: Remember how to complete the square and use substitution (and look at the notes and the part of the moment-generating function).