MTH/BIO 415 MIDTERM REVIEW SHEET
MIDTERM: Wednesday, October 23, 2013

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1. REVIEW ALL HOMEWORK ASSIGNMENTS ... thoroughly.

2. Consider the following discrete time population model for a single species

$$N_{t+1} = \frac{rN_t}{(N_t)^b}$$

   a) What is being modeled here? What is the population-dependent growth rate function? Plot this function using matlab for different values of $r$ and $b$

   a) Describe in words what the parameters $r$ and $b$ represent biologically.

   b) Find all steady state solutions to the model.

   c) Determine the stability of each steady state.

   d) Given $r = 6$ and $b = 1.5$, what would you expect the model to predict for the population. Sketch a graph of $N$ versus $t$.

   d) Given $r = 6$ and $b = 2.1$, what might you expect the model to predict for the population. Sketch a graph of $N$ versus $t$.

3. Consider the following simple matrix model for a certain species of turtle grouped into 3 yearly age classes.

$$\mathbf{X}_{n+1} = \begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = \begin{pmatrix} 0.90 & 0.35 & 0.00 \\ 0.00 & 0.00 & 0.25 \\ 0.30 & 0.00 & 0.00 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \mathbf{A}\mathbf{X}_n$$

   (a) Explain, in words, what you think the model is trying to represent. What are the $x, y, z$ variables representing. What does each term in the matrix represent?

   (b) A savvy student takes the matrix and computes its eigenvalues and eigenvectors using MatLab. She gets the following results:

```
>> [v,d] = eig(A)

v =
   0.9554            -0.0209 + 0.3499i  -0.0209 - 0.3499i
   0.0955            -0.1195 - 0.7893i  -0.1195 + 0.7893i
   0.2796             0.4892 - 0.0222i   0.4892 + 0.0222i


d =
   1.0250                  0                  0
        0            -0.0225 + 0.2135i        0
        0                  0            -0.0225 - 0.2135i
```

   Use these results to answer the following questions:

i. What does the model predict will happen to the turtle population as time goes on? Why?

ii. After a long time (say 100 years), what will be the distribution of turtles among age classes? (ie: What percentage of turtles will be in the middle ($y$) age class? What percentage will be in the old ($x$) age class?)

(c) This same, savvy student wonders how sensitive the matrix is to changes in parameters. She gets hold of some sensitivity code and computes:

```
>> [S,E] = sense(A)
S =

    0.9193    0.0919    0.2691
    0.4036    0.0403    0.1181
    0.1378    0.0138    0.0403

E =
    0.8790    0.0403         0
         0         0    0.0403
    0.0403         0         0
```

What does this all mean as far as the turtles go? Given the model and the sensitivity results, what is the most important biological concern for the continued health of the turtle species? EXPLAIN fully.

4. Write down a simple continuous time model for a single species population that grows *logistically*. Describe the dynamics of the model - fixed points, and stability.

5. Before most (but not all) of us were born, Beddington and May (1982) proposed the following model to study the interactions between baleen whales and their main food source (krill) in the Southern ocean:

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right) - axy \\ \frac{dy}{dt} &= sy\left(1-\frac{y}{bx}\right) . \end{aligned}$$

a) Explain the model in words. What does each equation model? Which are the whales, which are the krill?

b) Explain the parameters biologically. In particular, look at the $dy/dt$ equation as a logistic-growth model. What is the carrying-capacity here?

c) Find ALL steady-state, fixed point solutions for this system. Describe these in biological terms.