MTH/BIO 415 - Fall 2013

Homework Assignment #5: Nonlinear Differential Equations

Due: Monday, November 4

1. For the following first-order ordinary differential equations, sketch solutions by first sketching the given slope-field.

   (a) $\frac{dy}{dx} = y(y-a)(b-y), \quad b > a$

   (b) $\frac{dy}{dx} = (1 - \frac{y}{K})y$

   (c) $\frac{dy}{dx} = \frac{y}{x}$

   (d) $\frac{dy}{dx} = -\frac{x}{y}$

2. The following has been proposed as a model for the interaction between two species of fish (x) and (y).

$$\begin{aligned} \dot{x} &= rx\mathrm{e}^{-\beta x} - axy \\ \dot{y} &= (cx - b)y \ . \end{aligned}$$

   a) Explain, in words, what each term in the model is trying to describe. What do the parameters r, a, b, c and β (all positive numbers) represent?

   b) Rescale the equations to reduce the number of parameters.

   c) Find all the *null-clines* of the re-scaled model.

   d) Sketch the null-clines on the phase-plane.

   e) Find the fixed-points (equillibrium points) of the model. Indicate them on the graph.

   f) Determine the stability of the fixed points graphically - Sketch a few solutions in the phase plane.

   g) Explain, concisely in words, what the model predicts for the dynamics of the two species.

3. Consider a lake with some fish attractive to fishermen. Your task is to model the fish-fishermen interaction. That is, write a differential equation model for this system.

Fish Assumptions:

1) Fish grow logistically in the absence of fishermen.
2) The presence of fishermen depresses fish growth at a rate jointly proportional to the fish and fishermen population.

Fisherman Assumptions:

1) Fishermen are attracted to the lake at a rate directly proportional to the amount of fish in the lake.
2) Fishermen are discouraged from the lake at a rate directly proportional to the number of fishermen already there.

(a) Write down the model - Carefully explain what each of the parameters mean.

(b) Rescale the model to reduce the number of parameters.

(c) Sketch the null-clines and direction field in the fish-fisherman phase plane.

(d) Sketch some solution curves.

(e) What does the model say about the populations of fish and fishermen in this lake? How does this prediction depend on parameters?

(f) Suppose the Department of Fish and Game decides to stock the lake with fish at a constant rate: what changes would you make to your original model?