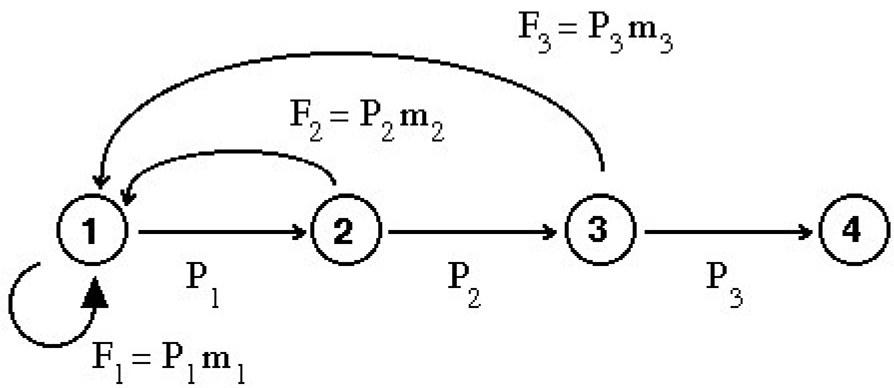
MTH/BIO 415 - Fall 2013

Homework Assignment #4: Matrix Models: Elasticity and Sensitivity

Due: Wednesday, October 9

1. Consider the follow age-structured population graph for female endangered black-footed ferrets:



(a) Ferrets start reproduction at the end of their first year. On average, first year breeders produce 1.5 female offspring, second year breeders produce 1.7 and third year breeders produce 1.75. Estimates of survival rates are 0.464, 0.534, and 0.346 for first three years respectively.

(b) /textitCLEARLY explain what is going on the graphical model of the ferret life-cycle. How long to ferrets live? What are the population classes being considered? Why is there a *self-loop* in the first age-class? What does the oldest group do?

(c) /textitCLEARLY explain why the fertilities (fecundities, F_i) have the form given. Think a bit. For the graph shown, when (in time) should one conduct censuses to estimate the parameters?

(d) Translate the life-cycle graph into a matrix model.

$$\mathbf{X}_{i+1} = \mathbf{A}\mathbf{X}_i$$

What does the discrete time unit, $i \to i+1$ represent in reality? What are the entries in the matrix $\mathbf{A}$?

(e) Write a MatLab script to numerically determine the time evolution of the female ferret population. Start from some initial condition, say: One female in each age class. Plot the ratio of one year-old ferrets to 2 year-old ferrets as a function of time. What happens to this ratio? Change the initial condition and redo. Does the ratio change? Plot the percent increase/decrease in population as a function of time. (In other words, plot Number(t+1)/Number(t) as a function of time.) What happens to this ratio as time increases?

(f) Use MatLab to find the eigenvalues and eigenvectors of the Matrix **A**. What do these objects say about the population? In particular, compare the structure of the eigenvector associated with the largest eigenvalue to what your model shows for the population structure. COMMENT.

(g) Use the matlab script `sensitivity.m` to find the sensitivity and elasticity matrices for **A**. What element of **A** is the eigenvalue most sensitive to? Does this element have a biological meaning? What non-zero element of **A** produces the largest proportional change in the eigenvalue?

(h) Explain, clearly, what this implies for conservation efforts of the ferret population.

2. Load the data in the file: `MatrixModels.mat` into MatLab. To do this, simply type:

```
>> load('MatrixModels.mat')
```

at the MatLab prompt. This should give you access to some matrices: `P` is a 10×10 model of the US human population based on 5 year age classes using best fit 1971 data. `W` is a 4×4 model of the Killer-Whale population from Brault and Caswell (Research paper available on the MathBio Website).

(a) You should be familiar enough now with matrix population models to understand what these folks are trying to do. For both models, rewrite the matrix model as an age-structured graph (like the one shown for the Ferrets). What is the essential difference between the Killer-Whale and Human models?

(b) We want to compare the two models:

i. What does each model predict for the population? Growth or Decay?

ii. Make a plot of the equilibrium structure of the population for each. (Use the number of first stage population for the comparisons ... ie: plot ration of number in each age class to number in first age class). Are the overall age structures of Killer-Whale and Human populations similar? Different? Explain.

iii. Do elasticity analysis of the two models. For each, plot the elasticity of the principal eigenvalue to Fecundities and Survivorships of each stage. Are the results similar or different for the Killer-Whales and Humans? Explain.