

**Material Covered:**

- Text: Determinants: Chapter 8: Sections 1,2,3
- Text: Eigen-Problems: Chapter 9: Sections 1,2,3.1
- Text: Systems of ODES: Chapter 10: Sections 1,2,3

DO ALL PROBLEMS IN SYLLABUS FOR ABOVE MATERIAL IN TEXT!!!

1. Write down as many statements as you can that indicate whether an  $n \times n$  matrix of real numbers is *non-singular*.
2. When is an  $n \times n$  matrix of real numbers *diagonalizable*?
3. Consider the following matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

- (a) Find  $|\mathbf{A}|$ . Does  $\mathbf{A}^{-1}$  exist?
  - (a) Use row reduction to show that  $\mathbf{A}$  is non-singular. ( $\mathbf{A}^{-1}$  exists)
  - (b) Find  $\mathbf{A}^{-1}$
  - (c) Show that  $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$
4. Consider the following matrix:

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ h & 2 \end{pmatrix}$$

where  $h$  is a real number.

- (a) For what values of  $h$  is the matrix NOT *invertible*?
  - (b) For what values of  $h$  is the matrix NOT *diagonalizable* ?
5. Find  $P$  and  $P^{-1}$  that diagonalize the matrix

$$A = \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & 0 \\ 0 & -1 & 3 \end{pmatrix}$$

Use the results to find  $A^5$ .

6. Consider the following second order ODE:

$$y'' + 2y' - 3y = 0$$

- (a) Find the general solution by reducing the equation to a system of first order equations.
- (b) Write the fundamental solution matrix.
- (c) Given  $y(0) = 0, y'(0) = 1$ , use the fundamental matrix to write the solution to the initial value problem.

7. Find the general solution to the following system of ODEs:

$$X' = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} X$$

Write the solution in terms of the fundamental matrix.

8. Find the general solution to the following system of ODEs:

$$X' = \begin{pmatrix} 0 & 1 \\ -5 & -6 \end{pmatrix} X$$

Write the solution in terms of the fundamental matrix.

9. Find the general solution to the following system of ODEs:

$$X' = \begin{pmatrix} 6 & -1 \\ 5 & 2 \end{pmatrix} X$$

Write the solution in terms of the fundamental matrix.

10. Find the general solution to the following system of ODEs:

$$X' = \begin{pmatrix} 4 & -4 \\ 1 & 0 \end{pmatrix} X$$

Write the solution in terms of the fundamental matrix.

11. Find the general solution to the following system of FORCED ODEs:

$$X' = \begin{pmatrix} 4 & \frac{1}{3} \\ 9 & 6 \end{pmatrix} X + \begin{pmatrix} -3 \\ 10 \end{pmatrix} e^t$$