Math 330 - Exam 2 - Spring 2018

NAME:

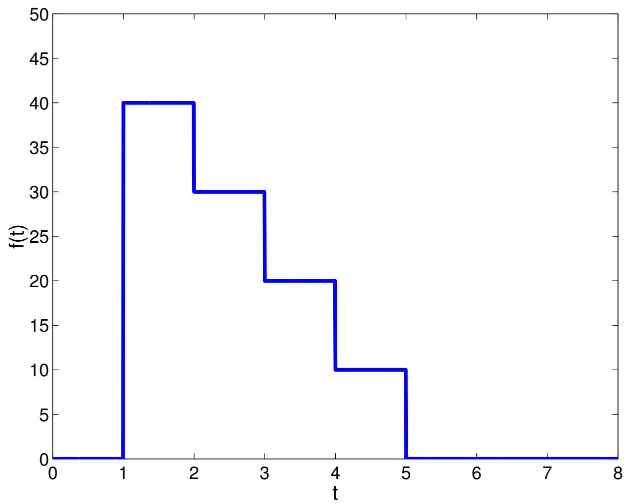
Please place your name on the top of the paper. Show your work in the space provided for each question. Partial credit can only be given for work shown (!). GOOD LUCK!

Question (1) (10 points)

For the forcing function shown below:

(a) Write $f(t)$ in terms of Heaveside Step Functions.

(b) Find the Laplace Transform $\mathcal{L}\left[f(t)\right](s)$



Question (2) (8 points)

Use properties of the Laplace transform to find the inverse transform of the following functions:

(a)

$$\frac{\mathrm{e}^{-\pi s}}{s^2+9}$$

(b) Write inverse transform as a convolution integral (no need to integrate!)

$$\left(\frac{1}{s^2}\right)\left(\frac{s}{s^2+4}\right)$$

Question (3) (12 points)

Use Laplace transform techniques to solve the following forced ODE initial value problem:

$$y' + 3y = f(t) = e^{4t}, \quad y(0) = y'(0) = 0.$$

EXTRA CREDIT: Using the Convolution Theorem, write an integral expression for the solution to the ODE for ANY reasonable $f(t)$.

Question (5) (10 points)

Use Laplace transform techniques to solve the following initial value problem:

$$y'' + 4y' + 68y = 0, \quad y(0) = 0, \;\; y'(0) = 1.$$

Question (6) (10 points)

Use Laplace transform techniques to solve the following initial value problem:

$$y'' + 4y' + 68y = \delta(t-10), \quad y(0) = 0, \;\; y'(0) = 0.$$

EXTRA CREDIT: What is the solution to:

$$y'' + 4y' + 68y = \delta(t), \quad y(0) = 0, \;\; y'(0) = -1.$$

If the ODE represents a mass with a spring and a damper, can you explain physically what is happening here?

Question (7) (10 points)

The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{pmatrix} a & b & c & d \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

1. Find the matrix $\mathbf{AB}$.
2. Find the matrix $\mathbf{BA}$.

Question (8) (10 points)

Let the set S consist of all vectors in R^3 on the plane $x - 2y + z = 0$.

(a) Explain why S is a subspace of R^3.

(b) Find a basis for S.

(c) Find the dimension of the subspace S.

(d) Is the vector $\mathbf{u} = \langle -3, 0, 3 \rangle$ in the span of the basis of S?

Question(9) (15 points)

Given the following matrix:

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 3 & 4 & 5 & 6 \\ 6 & 7 & 8 & 9 \end{pmatrix}$$

(a) Before doing anything, what is the maximum rank($\mathbf{A}$)?

(b) Use row reduction to the find the equivalent row reduced matrix $\mathbf{A}_r$.

(c) What is rank($\mathbf{A}$)?

(d) What is the dimension of the $Null - Space$ of $\mathbf{A}$?

(e) For $\mathbf{A}$ given above: Solve the homogeneous problem:

$$\mathbf{AX} = \mathbf{0}$$

Question(9a) (3 points) A fancy-pants mathematician makes the following claim about the matrix $\mathbf{A}$ defined in Question(9):

"$\mathbf{A}$ maps a p dimensional subspace of R^q to the zero vector in R^l."

What are the correct values of p, q and l for this to be a true statement?

Question(10) (12 points)

Consider the following system of equations:

$$\begin{pmatrix} 1 & -2 & 0 \\ h & 6 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ k \\ 1 \end{pmatrix}$$

1. Find all values of h and k for which the system has a UNIQUE solution.
2. Find all values of h and k for which the system has ∞-many solutions.
3. Find all values of h and k for which the system has NO solutions.