MTH 330 Exam 2 Review Sheet - Exam 2: Thursday March 29

Material Covered: Chapt 3: LaPlace Transforms 3.1-3.5, Chapt 10: Vectors 10.1-10.4, Chapt 11 11.1-11.4

1. Use the definition of the Laplace Transform to find the transform of the following functions (show all work):

   (a) $f(t) = 21 + 5t - 2e^{-\pi t}$

   (b)

$$f(t) = \begin{cases} 3, & 0 < t < 2 \\ 0, & 2 \leq t \leq 4 \\ -3, & t < 4 \end{cases}$$

2. State the two shifting theorems for Laplace Transforms.

   Use the shifting theorems to find the inverse transforms of

   (a)

$$F(s) = \frac{3}{s+4} e^{-\pi s}$$

   (b)

$$F(s) = e^{-8s} \frac{2s - 10}{(s^2 + 10s + 29)}$$

3. Find the solution to the following initial value problem using Laplace transform techniques.

$$y'' - 2y' + y = 0; \ \ y(0) = 3, \ y'(0) = 5$$

4. Solve the following forced ODE using transforms:

$$q' + 5q = f(t), \quad q(0) = 1$$

   where the forcing function is given by:

$$f(t) = \begin{cases} 1 & t < 3 \\ -1 & t \geq 3 \end{cases}$$

5. What is a convolution of two functions? What is the Laplace transform of a convolution product? (ie What is $\mathcal{L}[f * g]$)?

   Use the convolution theorem to find the inverse transforms of the following in terms of an integral. Do the integration if you can.

   (a)

$$T(s) = \left(\frac{4}{s+4}\right)\left(\frac{1}{s-6}\right)$$

   (b)

$$T(s) = \frac{3}{s(s^2-9)}$$

6. Graph the following functions:

$$f(t) = H(t+2) + H(t+1) - H(t-1) - H(t-2)$$

$$g(t) = H(t) - 2H(t-1) + 2H(t-2) - 2H(t-3) + H(t-4)$$

   Find $\int_{-\infty}^{\infty} f(t)dt$ and $\int_0^{\infty} g(t)dt$

7. Find the solution to the following initial value problem using Laplace transform techniques.

$$y'' + 2y' + 65y = \mathrm{e}^{-\mathrm{t}};\;\; \mathrm{y}(0) = 1,\; \mathrm{y}'(0) = 0$$

8. Find the solution to the following initial value problem using Laplace transform techniques.

$$y'' + 64y = \delta(t-2);\;\; y(0) = 0,\; y'(0) = 0$$

Vectors, Vector Spaces, Matrices

1. Find the angle between the following pairs of vectors. Are the vectors orthogonal?

$$\mathbf{u} = (1, 2, 3), \mathbf{v} = (1, -2, 1); \qquad \mathbf{u} = (1, 2, 3, 1), \mathbf{v} = (1, 0, 1, 0)$$

2. Find the projection of **v** on **u**:

$$\mathbf{u} = (1, -1, 4), \mathbf{v} = (-3, 2, -1)$$

3. Prove the Cauchy-Schwarz Inequality:

$$|\mathbf{F} \cdot \mathbf{G}| \leq \|\mathbf{F}\|\|\mathbf{G}\|$$

4. State whether the following are proper subspaces of R^5. Clearly explain why.

(a) The set of all vectors in R^5 with first element $= 1$.

(b) The set of all vectors which are scalar multiples of

$$v = (1, 0, -3, 2, -2)$$

(c) All vectors, $\mathbf{F}$ in R^5 such that $||\mathbf{F}|| \leq 1$

5. S is the set of all vectors in R^4 of the form $(x, y, x - y, 3x + 2y)$.

   Show that the set S is a subspace of R^4. Find a basis for this subspace and the dimension of the subspace.

6. Given:

$$A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \text{ and B} = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$$

   Determine which of the following matrix products are defined and, for those that are defined, compute the product: (i) AB (ii) BA (iii) $A^T B$

7. In each problem: find A_r, the row reduced echelon form of the matrix and the *product matrix* Ω such that $\Omega A = A_r$. Find the rank of each matrix.

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \quad A = \begin{pmatrix} 1 & -1 & 3 & 4 \\ 1 & 0 & 1 & -1 \end{pmatrix}; \quad A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 0 \\ 1 & 0 \end{pmatrix}$$

8. For each of the matrices A defined in the last question, find the solution, X, to the homgeneous problem:

$$AX = \mathbf{0}$$

   For each, what is the dimension of the *null-space* of A?

9. Decide whether the following vectors form a basis for R^3:

   (a)

$$v_1 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

   (b)

$$v_1 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

10. Find the general solution to the following system of equations:

$$\begin{aligned} 8x_2 - 4x_3 + 10x_6 &= 1 \\ x_3 + x_5 - x_6 &= 2 \\ x_4 - 3x_5 + 2x_6 &= 0 \end{aligned}$$

11. Consider the following system of equations:

$$\begin{aligned} x + y - z &= 3 \\ x - y + 3z &= 4 \\ x + y + (K^2 - 10)z &= K \end{aligned}$$

(a) Find value(s) of K for which there are NO solutions.

(b) Find value(s) of K for which there are infinitely many solutions.

(c) Find value(s) of K for which there are unique solutions.

12. Consider the following set of linear equations:

$$\begin{aligned} x_1 - 3x_2 &= 1 \\ 2x_1 - hx_2 &= k \end{aligned}$$

(a) Find value(s) of h for which a UNIQUE solution exists for all k.

(b) Find value(s) of h and k for which NO solutions exist.

(c) Find value(s) of h and k for which there are an infinite number of solutions.