

10.2

Setup amounts to $Y \sim \text{Binomial}(20, p)$ to be tested with $H_0: p = .8$ vs. $H_1: p < .8$

RR is $\{y \leq 12\}$

a) Type I is reject in error is $p = .8$ but $y \leq 12$.

c) Type II is accept in error is $p < .8$ but $y > 12$.

$$b) \alpha = P(\text{Type I}) = P(y \leq 12 | p = .8) = \sum_{y=0}^{12} P(y | .8) = \text{pbinom}(12, 20, .8).$$

$$d) \beta(.6) = P(\text{Type II}) = P(y > 12 | p = .6) = \sum_{y=13}^{20} P(y | .6) =$$

$$= \text{pbinom}(13, 20, .6, \text{lower.tail} = \text{FALSE})$$

$$e) \text{pbinom}(13, 20, .4, \text{lower.tail} = \text{FALSE})$$

10.3

a) To find a rejection region we could either tabulate all x and pick a value, or we could use the `qbinom` function in \mathcal{R} .

c	10	11	12	13
α	.00259	.00998	.0321	.0867

So $\{y \leq 11\}$ would work.

Alternative: `qbinom(p)` returns the smallest F such that $F(x) \geq p$. Since we will want - at most p -, we will instead need `qbinom(p) - 1`.

b) `pbinom(11, 20, .6, lower.tail = FALSE)`

c) `qbinom(11, 20, .4, lower.tail = FALSE)`

10.19

V specified as 130V.

40 readings gave $\bar{V} = 128.6$ and $s = 2.1$.

$$H_0: V = 130$$

$$H_A: V < 130$$

$$\alpha = .05.$$

$$\text{By hand: } z = \frac{\bar{V} - V_0}{s/\sqrt{n}} = \frac{128.6 - 130}{2.1/\sqrt{40}}$$

Reject if smaller than $q_{\text{norm}}(.05) \approx -1.645$

... better: use t-test and reject if smaller than $qt(.05, 39) \approx -1.685$.

In R: use the BSDA package and the function

BSDA::tsum.test(128.6, 2.1, 40, mu = 130, alternative = "lower")

10.37

$$V_0 = 130$$

$$V_A = 128$$

$$N = 40$$

$$S = 2.1$$

$\beta(128)?$

Reject if $z < -1.645$

Accept if $z \geq -1.645$

Rejection region is equivalent to rejecting if

$$\frac{\bar{V} - 130}{2.1/\sqrt{40}} < -1.645$$

ie

$$\bar{V} < 130 - 1.645 \cdot 2.1/\sqrt{40} \approx 129.45$$

$$\begin{aligned} \text{So } \beta(128) &= P(\bar{V} \geq 129.45 \mid V = 128) = \\ &= \text{pnorm}(129.45, \text{mean} = 128, \text{sd} = 2.1/\sqrt{40}, \text{lower.tail} = \text{FALSE}) \end{aligned}$$

10.49

Upper confidence bound is given as $\bar{V} - z_{\alpha/5}/\sqrt{n} =$
 $= 128.6 + 1.645 \cdot 2.1/\sqrt{40} \approx 129.146.$