Differential Geometry, Mth 445

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The development of calculus by Newton and Leibniz in the 17th century opened up the possibility to study more complicated curves and surfaces. This has been very useful in engineering, science, and architecture.

For instance, what is the shape that a chain assumes under its own weight when it is supported only at its ends ?

Figure 1: catenary

Galileo thought the curve was a parabola. Thanks to the tools of calculus, we will discover that such statement is incorrect. This curve, known as "catenary", has been applied in the construction of the arches that you can see in some cathedrals (St Paul Cathedral in London), or in some of the beautiful houses built by Gaudi in Barcelona.

The Swiss mathematician Euler was a very prolific mathematician who investigated the analytical properties of many curves. Among others, one of these curves (named "Euler spiral") is precisely the shape that is used nowadays in the construction of highway ramps, or in some roller coasters. We will see why, and what geometrical concepts led to design such shapes.

Those ideas can be extended to study properties of surfaces. Take for instance a piece of wire, connect the two ends together, and dip it in a bath of soapy water. The thin membrane that spans the wire boundary is called a "minimal surface". Euler realized that by rotating the catenary one obtains an example

of such surface called the catenoid. We will see how to compute many other examples of such "minimal surfaces".

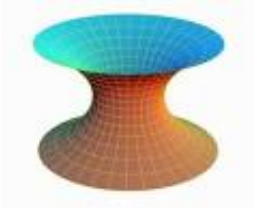

Figure 2: catenoid

The key concept is the notion of curvature, which has been studied in details by Gauss, who applied it in a very surprising and clever way in cartography.

Figure 3: Gauss (1777-1855)

The contributions by Gauss and by his student Riemann opened up a new era in mathematics, that is fundamental in the development of the general relativity, and contemporary physics.

Time and Place Monday: 2:30–4:10pm, 1S-114
Wednesday: 2:30–4:10pm, 1S-114