Recent developments in minimal surfaces

Titles and Abstracts

The Graduate Center, CUNY
365 Fifth Avenue
New York, NY 10016
Science Center, Room 4102

Thursday, February 23rd, 2012
9:00am till 4:00pm

9:30–10:00:	*Coffee*
10:00–11:00:	William P. Minicozzi
11:15–12:15:	Mike Wolf
12:15–1:30:	*Lunch*
1:30–2:30:	Jean-Marc Schlenker
2:45–3:45:	William Meeks
3:45–4:15:	*Discussion*

William Meeks III, University of Massachusetts

Constant mean curvature spheres in homogeneous three dimensional manifolds

Using the classical holomorphic quadratic Hopf differential, Hopf proved that a constant mean curvature $H > 0$ sphere in three-dimensional Euclidean space is a round sphere of radius $1/H$. In particular, the moduli space of such spheres up to congruence is parametrized by the mean curvatures that lie in the interval $(0, \infty)$ and every such sphere has index one for the stability operator. In recent years, this result of Hopf has been generalized to some other simply connected homogeneous 3-manifolds X with 4 dimensional isometry group such as the Riemannian product of a round sphere with the real number line $\mathbb{R}$. I will discuss the final version of this classification result due to Meeks, Mira, Perez and Ros. In my talk I will focus on the case where X is a metric Lie group (a Lie group with a left invariant metric). When X is diffeomorphic to R^3, these spheres of constant mean curvature H are parametrized by their values of H in the open interval $(2Ch(X), \infty)$, where $Ch(X)$ is the Cheeger constant of X, and we show how to calculate $Ch(X)$ in terms of the metric Lie algebra of X. When X is diffeomorphic to the 3-sphere, then we prove that it admits for every H greater than or equal to 0, a unique sphere $S(H)$ of constant mean curvature H, and this sphere has index one and is Alexandrov embedded (in general when H is not equal to 0, $S(H)$ may possibly not be embedded for certain homogeneous metrics). Also when X is diffeomorphic to the 3-sphere, the minimal sphere $S(0)$ in X is embedded and contains 3 geodesics of rotational symmetry that meet orthogonally on $S(0)$, and so $S(0)$ always separates X into isometric regions.

William Minicozzi II, Johns Hopkins University
Dynamics and singularities of mean curvature flow

Mean curvature flow (MCF) is a nonlinear heat equation where the hypersurface evolves to minimize its surface area. Minimal surfaces are static solutions of MCF. If we start the flow at any closed hypersurface in R^n, then singularities must develop. The major problem is to understand the possible singularities and the behavior of the flow near a singularity. I will survey work on this with Toby Colding.

Jean-Marc Schlenker, Université Toulouse III
Maximal surfaces in the anti-de Sitter space and the universal Teichmüller space

A homeomorphims of the circle is quasisymmetric if it extends to a quasiconformal diffeomorphism of the disk. We prove that it then has a unique quasiconformal extension to the hyperbolic disk which is minimal Lagrangian (it is area-preserving and its graph is minimal). The proof is based on an existence and uniqueness result for maximal surfaces in the anti-de Sitter space spanning a given space-like curve at infinity. Joint work with Francesco Bonsante (Pavia).

Michael Wolf, Rice University
Teichmüller theory and minimal surfaces

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