Multiple solutions of some differential systems with strongly indefinite variational structure

Abstract

Consider the following differential systems

$$\begin{cases} -\Delta u = g(x,v) \text{ in } \Omega \\ -\Delta v = f(x,u) \text{ in } \Omega \\ u = v = 0 \text{ on } \partial\Omega \end{cases}$$

$$\begin{cases} \partial_t u - \Delta_x u + V(x)u = H_v(t,u,v) \ \text{ in } \mathbb{R}\times\Omega \\ -\partial_t v - \Delta_x v + V(x)v = H_u(t,u,v) \ \text{ in } \mathbb{R}\times\Omega \\ z(t,x) = z(t+T,x) \quad \forall (t,x) \in \mathbb{R}\times\Omega \\ z(t,x) = 0 \quad \forall (t,x) \in \mathbb{R}\times\Omega \end{cases}$$

where Ω is bounded domain in $\mathbb{R}^N$, f, g and H are nonlinear functions. Solutions of these systems are critical points a functional of the form

$$\Phi(u) = \frac{1}{2}\big\langle Lu, u\big\rangle + N(u)$$

defined on a Hilbert space X, where $L : X \to X$ is a bounded selfadjoint linear operator, N a nonlinear operator and the quadratic part $\big\langle Lu, u\big\rangle$ has infinitely many positive and infinitely many negative eigenvalues. Such a functional is said to be strongly indefinite, and its study gives rise to an interesting problem because the usual critical point theorems cannot be applied. By using some generalizations of the symmetric mountain pass theorem we will prove that these systems have infinitely many solutions. In the cases we consider, we do not know whether the usual compactness condition (Palais-Smale condition) holds.

(Joint work with Fabrice Colin and Tomasz Kaczynski)

Cyril Joel Batkam
Department of Mathematics
University of Sherbrooke