MTH 372 Practical Machine Learning, Spring 2026 **HW 1**

Name: ____

(1) Let $T\colon V \to W$ be a linear map between two vector spaces. Show that $\dim(\ker(T)) + \dim(\mathrm{Im}(T)) = \dim(V)$.

(2) Consider the set of all polynomials in one variable with real coefficients of degree less than or equal to three.
  (a) Show that this set forms a vector space of dimension four.
  (b) Find a basis for this vector space.
  (c) Show that differentiating a polynomial is a linear transformation.
  (d) Given the basis chosen in part (b), write down the matrix representative of the derivative.

(3) Denote the vector space of all functions $f\colon \mathbb{R} \to \mathbb{R}$ which are infinitely differentiable by $C^\infty(\mathbb{R})$. This space is called the space of smooth functions.
  (a) Show that $C^\infty(\mathbb{R})$ is infinite dimensional.
  (b) b. Show that differentiation is a linear transformation:

$$\frac{d}{dx}\colon C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R}).$$

  (c) For a real number λ, find an eigenvector for $\frac{d}{dx}$ with eigenvalue λ.

(4) Show that the set M_2 of 2×2 matrices is a vector space.
  (a) Write down an explicit basis.
  (b) Show that the set of symmetric matrices is a subspace and find its dimension.
  (c) Show that the map $A \mapsto A - A^T$ is linear, and write down the matrix for this map using your basis from (a).
  (d) What is the kernel of this map?