Example $\frac{d}{dx}\int_0^{x^2} \cos(t)\,dt$ ← use chain rule!

§5.6 Substitution / change of variable "reverse chain rule for integration"

recall: chain rule: $\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\,g'(x)$.

substitution / change of variable

$\int f(x)\,dx$, $x(u)$ $\displaystyle\int_{x=a}^{x=b} f(x)\,dx = \int_{u=x^{-1}(a)}^{u=x^{-1}(b)} f(x(u))\frac{dx}{du}\,du$

3 things to change: function, differential, limits

mnemonic: cancelling fractions $dx = \frac{dx}{du}\,du$

Q: why does this work?

$\int f(x)\,dx$, $x(u)$, set $F(x) = \int f(x)\,dx$, so $F'(x) = f(x)$

$\int f(x)\,dx = F(x)$, differentiate wrt to u: $\frac{d}{du}\left(F(x(u))\right) = F'(x(u))\,x'(u)$ ⊕

$\int f(x)\,dx =$ integrate ⊕ wrt u $= \int f(x(u))\frac{dx}{du}\,du$.

useful fact: $\frac{du}{dx} = 1\Big/\frac{dx}{du}$

Examples ① $\displaystyle\int_0^1 e^{-7x}\,dx$ set $u = -7x$ $\frac{du}{dx} = -7$

$\displaystyle\int_0^{-7} e^u \frac{dx}{du}\,du = \int_0^{-7} e^u \cdot -\frac{1}{7}\,du = -\frac{1}{7}\int_0^{-7} e^u\,du = -\frac{1}{7}\left[e^u\right]_0^{-7} = -\frac{1}{7}(e^{-7}-1)$

② $\displaystyle\int_0^2 x^2\sqrt{x^3+1}\,dx$, $\displaystyle\int_0^{\pi/4} \tan^3 x \sec^2 x\,dx$, $\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx$

§5.8 More integrals

recall: $\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}$ so $\int \frac{1}{x}\,dx = \ln|x| + c$