Example what shape of cylindrical can minimizes the surface area? if you want total volume to be $1\,ft^3$?

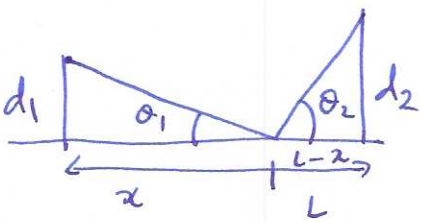
$V = \pi r^2 h \Rightarrow h = \dfrac{V}{\pi r^2}$.

$A = 2\pi r^2 + 2\pi r h$

$A = 2\pi r^2 + 2\pi r \dfrac{V}{\pi r^2} = 2\pi r^2 + \dfrac{2}{r}$ $\quad V = 1$.

$A'(r) = 4\pi r - \dfrac{2}{r^2}$ solve $A'(r) = 0$: $r^3 = \dfrac{1}{2\pi}$ $\quad r = \sqrt[3]{1/2\pi}$.

Example a ball bounces off a wall. Show that the path which minimizes distance has equal angles of incidence and reflection.

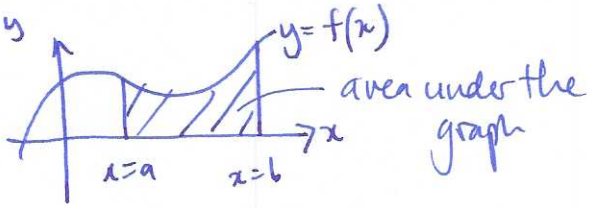


$f(x) =$ distance $= \sqrt{d_1^2 + x^2} + \sqrt{d_2^2 + (L-x)^2}$

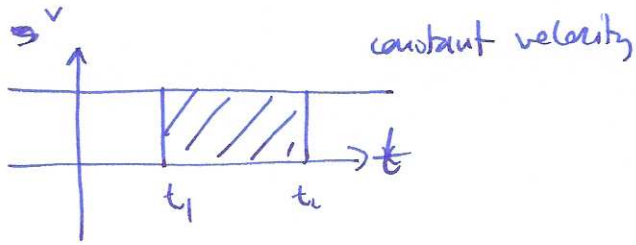
$f'(x) = \frac{1}{2}\left(d_1^2 + x^2\right)^{-1/2} \cdot 2x + \frac{1}{2}\left(d_2^2 + (L-x)^2\right)^{-1/2} \cdot 2(L-x) \cdot (-1)$.

$f'(x) = 0$: $\dfrac{x}{\sqrt{d_1^2 + x^2}} = \dfrac{L-x}{\sqrt{d_2^2 + (L-x)^2}}$ $\Leftrightarrow$ $\cos\theta_1 = \cos\theta_2 \Rightarrow \theta_1 = \theta_2$.

§5.1 Approximating area

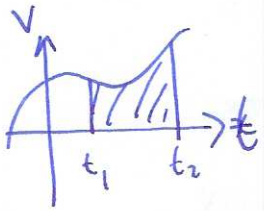


Example



distance travelled = velocity × time
= area under the graph

non-constant velocity: fact: distance travelled = area under the graph.



finding the area: approximate by rectangles