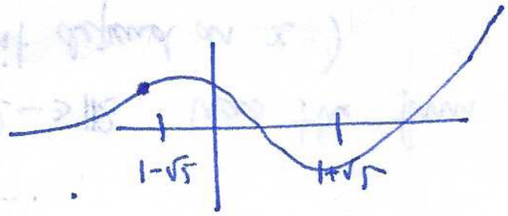
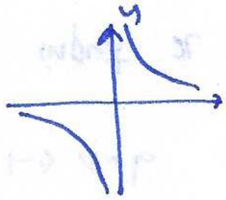
$f'(x) = (4+2x-x^2)e^{x}$

$f''(x) = (6-x^2)e^{x}$



Asymptotes

Example : $f(x) = \frac{1}{x}$



$y=0$ horizontal ~~vertical~~ asymptote

$x=0$ vertical asymptote

Defn $x=c$ is a vertical asymptote if $\lim_{x\to c^{\pm}} f(x) = \pm\infty$.

$y=c$ is a horizontal asymptote if $\lim_{x\to+\infty} f(x) = c$ or $\lim_{x\to-\infty} f(x) = c$

observation : rational functions $\frac{p(x)}{q(x)}$ have horizontal asymptotes if $\deg p \leq \deg q$

Example $f(x) = \frac{x^2+x+1}{3x^2+2} \sim \frac{x^2}{3x^2} \to \frac{1}{3}$ as $x \to \pm\infty$.

Example sketch graph of $f(x) = \frac{3x+2}{2x-4}$

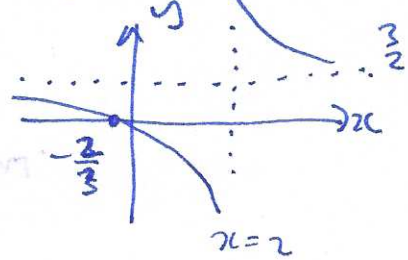
① find vertical asymptotes $\leftrightarrow$ denominator zero : $2x-4=0 \Rightarrow x=2$

② find $f'(x) = \frac{(2x-4)\cdot 3 - (3x+2)\cdot 2}{(2x-4)^2} = \frac{-16}{(2x-4)^2} = \frac{-4}{(x-2)^2} \Leftarrow$ always $-$ve.

so f decreasing, no critical points except at vertical asymptote $x=2$

③ $f''(x) = \frac{8}{(x-2)^3}$ +ve for $x>2$ concave up

−ve for $x<2$ concave down



④ horizontal asymptotes $\lim_{x\to+\infty} \frac{3x+2}{2x-4} = \lim_{x\to+\infty} \frac{3}{2} = \frac{3}{2}$

behaviour near asymptote / zero:

		$-2/3$		2	
$3x+2$	$-$		$+$		$+$
$2x-4$	$-$		$-$		$+$
$f(x)$	$+$		$-$		$+$

<u>Example</u> sketch graph of $f(x) = \dfrac{x}{\sqrt{x^2+1}} = x(x^2+1)^{-1/2}$

① vertical asymptotes : none

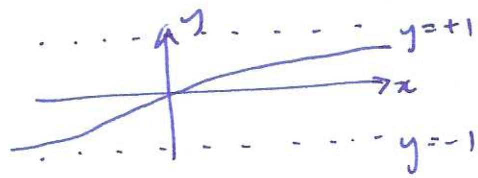
② find $f'(x) = \dfrac{\sqrt{x^2+1}\cdot 1 - x\cdot\frac{1}{2}(x^2+1)^{-1/2}\cdot 2x}{x^2+1} = \dfrac{x^2+1-x^2}{(x^2+1)^{3/2}} = (x^2+1)^{-3/2} > 0$

$\Rightarrow$ f increasing, no critical points

③ $f''(x) = -\frac{3}{2}(x^2+1)^{-5/2}\cdot 2x$ inflection point at $x=0$ +ve for $x<0$, −ve for $x>0$

④ horizontal asymptotes $\lim\limits_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}} \overset{l'H}{=} \lim\limits_{x\to\infty} \dfrac{1}{\frac{1}{2}(x^2+1)^{-1/2}\cdot 2x} = \lim\limits_{x\to\infty} \sqrt{1+1/x^2} = 1$

$\lim\limits_{x\to-\infty} \dfrac{x}{\sqrt{x^2+1}} = \lim\limits_{x\to\infty} \dfrac{-x}{\sqrt{x^2+1}} = -1$

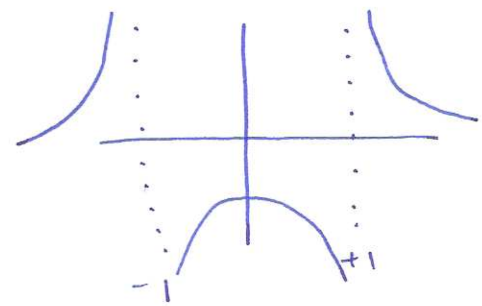


<u>Example</u> $f(x) = \dfrac{1}{x^2-1} = \dfrac{1}{(x-1)(x+1)}$

① vertical asymptotes at $x = \pm 1$

② $f'(x) = -(x^2-1)^{-2}\cdot 2x$ critical points at $x=0$

③ $f''(x) = \dfrac{6x^2+2}{(x^2-1)^3}$



④ etc...

§4.7 Optimization

<u>Example</u> a piece of wire of length L is bent into a rectangle. what is the largest area of the rectangle?

$\square\, y$, x area $\left.\begin{array}{l} A = xy \\ L = 2x+2y \end{array}\right\}$ $y = \frac{L}{2} - x$

$A = x\left(\frac{L}{2} - x\right) = \frac{L}{2}x - x^2$

$A'(x) = \frac{L}{2} - 2x$ $x = \frac{L}{4}$ (local max) so max occurs when $x = y = \frac{L}{4}$

area is $\frac{L^2}{16}$.