③ $\lim_{x\to 1} \frac{x^{100}-1}{x-1} = \lim_{x\to 1} \frac{100x^{99}}{1} = 100$

③ $\lim_{x\to \frac{\pi}{2}} \frac{\cos^2 x}{\sin x - 1} = \lim_{x\to\frac{\pi}{2}} -\frac{2\cos x \sin x}{\cos x} = \lim_{x\to\frac{\pi}{2}} -2\sin x = -2$

④ $\lim_{x\to 0+} x\ln x = \lim_{x\to 0} \frac{\ln(x)}{1/x} = \lim_{x\to 0} \frac{1/x}{-1/x^2} = \lim_{x\to 0} -x = 0$

⑤ $\lim_{x\to 0} \frac{e^x - x - 1}{\cos x - 1} = \lim_{x\to 0}\frac{e^x - 1}{-\sin x} = \lim_{x\to 0} \frac{e^x}{-\cos x} = -1$

⑥ $\lim_{x\to 0} \frac{1}{\sin x} - \frac{1}{x} = \lim_{x\to 0} \frac{x - \sin x}{x\sin x} = \lim_{x\to 0} \frac{1 - \cos x}{\sin x + x\cos x} = \lim_{x\to 0} \frac{\sin x}{\cos x + \cos x + x\sin x} = 0$

⑦ $\lim_{x\to 0} x^x = \lim_{x\to 0} e^{x\ln x} = e^{\lim_{x\to 0} x\ln x} = e^0 = 1.$

Comparing growth rates of functions

Q: which grows faster $(\ln(x))^2$ or $\sqrt{x}$?

$\lim_{x\to\infty} \frac{\sqrt{x}}{(\ln(x))^2} = \lim_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{2\ln(x)\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{x}}{4\ln(x)} = \lim_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{4/x} = \lim_{x\to\infty} \frac{\sqrt{x}}{8} = \infty$

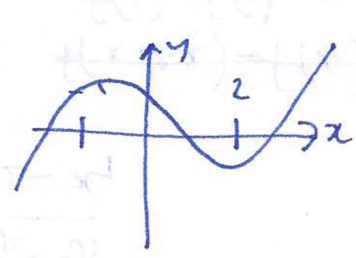
so $\sqrt{x}$ grows faster.

Thm e^x grows faster than any polynomial.

Proof $\lim_{x\to\infty} \frac{e^x}{x^n} = \lim_{x\to\infty} \frac{e^x}{nx^{n-1}} = \lim_{x\to\infty} \frac{e^x}{n(n-1)x^{n-2}} = \cdots = \lim_{x\to\infty} \frac{e^x}{n!} = \infty.$ □.

§4.6. Graph sketching and asymptotes

Example① sketch graph of $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 3$



useful info: • critical points $f'(x) = 0$: $f'(x) = x^2 - x - 2$
$(x-2)(x+1)$

- sign of $f'(x)$
- sign of $f''(x) = 2x - 1$

critical points at $x = -1, 2$ $\quad f''(-1) = -3 < 0$ local max

$f''(2) = 3 > 0$ local min

② $f(x) = (4x - x^2)e^x$ critical points at $x = 1 \pm \sqrt{5}$