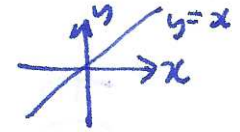
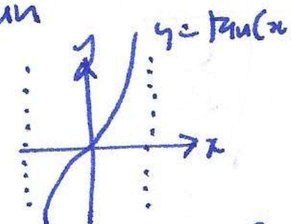
<u>warning</u>: some functions do not have any local max/min

<u>Examples</u> $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x$



$f: (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}$, $x \mapsto \tan x$



<u>Thm</u> If $f(x)$ is continuous on a <u>closed and bounded</u> interval, then $f(x)$ has an absolute max and an absolute min.

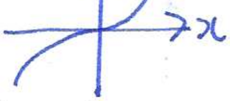
<u>Defn</u> $f(x)$ has a <u>local max</u> at $x=c$ if there is a small interval containing c such that $f(x)$ is an absolute max on this interval.

$f(x)$ is a <u>local min</u> at $x=c$ if there is a small interval containing c such that $f(c)$ is an absolute min on this interval.

<u>Defn</u> we say that $x=c$ is a <u>critical point</u> if $f'(c)=0$ (or undefined)

<u>warning</u> $f'(c)=0 \not\Rightarrow c$ is a local max or min.

<u>Example</u> $y = x^3$



$f'(x) = 3x^2$ $f'(0)=0$ but $x=0$ not local max or min.

• How to find the absolute max or min of a differentiable function on a closed interval $[a,b]$

① find critical points

② evaluate function at critical points <u>and</u> the endpoints.

<u>Examples</u> ① find abs max/min of $2x^3 - 15x^2 + 24x + 7$ on $[0,3]$

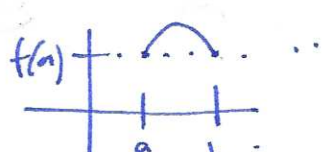
② $x^2 - 8$ on $[1,4]$

③ $\sin(x)\cos(x)$ on $[0,\pi]$

<u>Thm</u> (Rolle's Thm) suppose $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b)

If $f(a) = f(b)$, then there is a $c \in (a,b)$ s.t. $f'(c)=0$

<u>Proof</u>



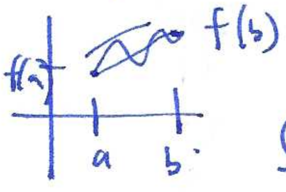
• if there is a local max/min $c \in (a,b)$ then $f'(c)=0$

• if no local max/min then $f(x) = \text{const} = f(a) = f(b)$

so $f'(c) = 0$ for all $c \in (a,b)$. $\square$

§4.3 First derivative test

Thm (MVT) (Mean Value Theorem) suppose f is cts on $[a,b]$ and differentiable on (a,b). Then there is a $c \in (a,b)$ s.t. $f'(c) = \frac{f(b)-f(a)}{b-a}$, i.e. there is a point where the slope of the tangent line is equal to the average rate of change over the interval.



Proof (Rolle's theorem turned on its side) □.

Corollary If $f(x)$ is differentiable and $f'(x)=0$ then $f(x)=c$ constant.

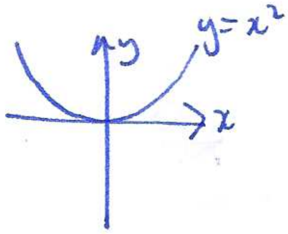
Proof suppose there is $a \neq b$ with $f(a) \neq f(b)$, then there is $c \in (a,b)$ with $f'(c) = \frac{f(b)-f(a)}{b-a} \neq 0$ ※ □.

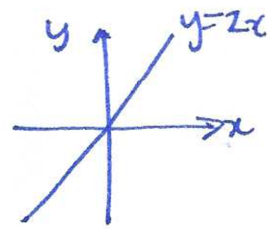
Monotonicity suppose $f(x)$ is differentiable on (a,b):

If $f'(x) > 0$ for all $x \in (a,b)$ then f is increasing on (a,b)

If $f'(x) < 0$ for all $x \in (a,b)$ then f is decreasing on (a,b)

Example $f(x) = x^2$ $\quad f'(x) = 2x$ $\quad f$

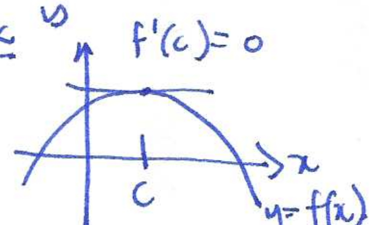


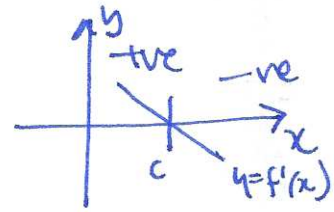


increasing on $(0,\infty)$
decreasing on $(-\infty,0)$

First derivative test

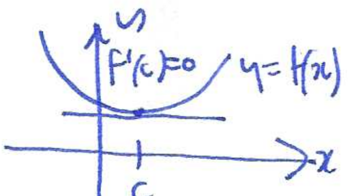
local max

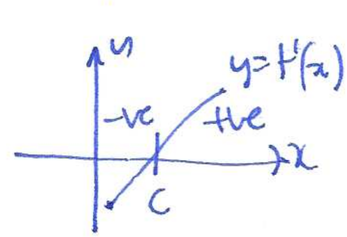




if $f'(x)$ goes from positive to negative at c
$\Rightarrow$ c is a local max

local min





if $f'(x)$ goes from negative to positive at c
$\Rightarrow$ c is a local min.