check $(\sinh(x))' = \left(\frac{e^x - e^{-x}}{2}\right)' = \frac{e^x + e^{-x}}{2} = \cosh(x)$

$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$ so $(\tanh(x))' = \frac{1}{\cosh^2(x)}$ etc.

inverse functions: $\frac{d}{dx}\left(\sinh^{-1}(x)\right) = \frac{1}{\sqrt{x^2+1}}$ $\frac{d}{dx}\left(\cosh^{-1}(x)\right) = \frac{1}{\sqrt{x^2-1}}$

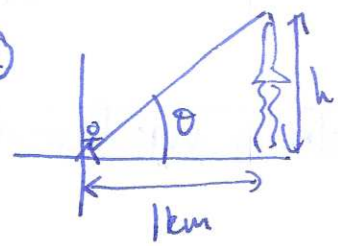
$\frac{d}{dx}\left(\tanh^{-1}(x)\right) = \frac{1}{1-x^2}$

§3.10 Related rates

Example ① Balloon  $V = \frac{4}{3}\pi r^3$, inflate balloon $V(t) = \frac{4}{3}\pi\left(r(t)\right)^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ so if $\frac{dV}{dt} = 1\,ft^3/min$ (inflate at constant rate)

and $r = 1ft$, then $1 = 4\pi \frac{dr}{dt}$ so $\frac{dr}{dt} = \frac{1}{4\pi}\,ft/min$

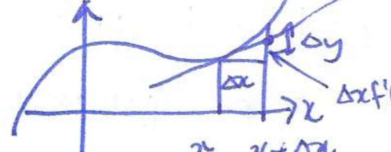
② 

if angle is $\theta = \frac{\pi}{3}$ and rate of change of angle is $\frac{d\theta}{dt} = \frac{1}{2}\,rad/\cancel{min}\,sec$

how fast is the rocket going?

$\frac{h}{1} = \tan(\theta)$ $\frac{dh}{dt} = \sec^2\theta \frac{d\theta}{dt}$ $\frac{dh}{dt} = \sec^2\left(\frac{\pi}{3}\right)\frac{1}{2} \simeq 0.1\,km/\cancel{min}\,sec$

§4.1 Linear approximation



if $f(x)$ is differentiable at x, and Δx is small, then $f(x+\Delta x) \simeq f(x) + f'(x)\Delta x$

so change in f: $\Delta f \simeq f(x+\Delta x) - f(x)$

$\simeq f'(x)\Delta x$

Example estimate $\sqrt{103}$. know: $\sqrt{100} = 10$

$f(x) = \sqrt{x} = x^{1/2}$ $\quad f(100) = 10$ $\quad$ so $\Delta f \approx f'(x)\,\Delta x$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$ $\quad f'(100) = \frac{1}{20}$ $\quad \frac{1}{20}\cdot 3$

so $\sqrt{103} \approx 10 + \frac{3}{20} = 10.15$

Example you make an 18" pizza. If the diameter is accurate to ± 0.4in, how much pizza do you gain or lose?

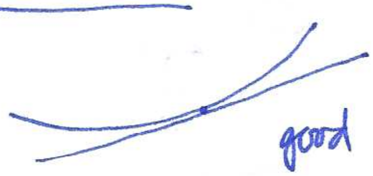
$A = \pi r^2$, $2r = D$ $\quad A = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}$, $A'(D) = \frac{2\pi D}{4} = \frac{\pi D}{2}$

$\Delta A = A'(18)\cdot\Delta D = \frac{1}{2}\pi \cdot 18 \cdot 0.4 \approx 11 \text{ in}^2$

Q: is this good or bad? absolute error $= 11$

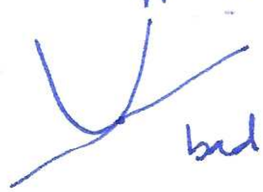
percentage error $= \left|\frac{\text{absolute error}}{\text{actual value}}\right| \times 100 = \frac{11}{\pi \cdot 18^2/4} \times 100 \sim 4\%$.

observation: when is the linear approximation a good approximation?



good

$|f''(x)|$ small



bad

$|f''(x)|$ large

§4.2 Extreme values

suppose $f(x)$ is defined on a closed interval $[a,b]$

Defn $f(c)$ is the absolute max if $f(c) \geq f(x)$ for all $x \in [a,b]$

$f(c)$ is the absolute min if $f(c) \leq f(x)$ for all $x \in [a,b]$

note: Q: where is the local/abs max/min? $\leftarrow$ want x value

Q: what is the local/abs max/min? $\leftarrow$ want y value.