Math 231 Calculus 1 Spring 26 Midterm 3b Part 1

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a US Letter page of notes but no calculator.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) Use l'Hôpital's rule to find $\lim_{x\to\infty} \frac{\ln(x)}{\sqrt[4]{x}}$.

$$\overset{l'H}{=} \lim_{x\to\infty} \frac{1/x}{\frac{1}{4}x^{-3/4}} = \lim_{x\to\infty} 4x^{-1/4} = 0$$

(2) Use l'Hôpital's rule to find $\lim_{x\to 0} \dfrac{\cos(2x)-1}{1-e^{-3x^2}}$.

$$\overset{l'H}{=} \lim_{x\to 0} \frac{-2\sin(2x)}{-e^{-3x^2}\cdot(-6x)} \overset{l'H}{=} \lim_{x\to 0} \frac{-4\cos(4x)}{-e^{-3x^2}\cdot 36x^2 + -e^{-3x^2}\cdot(-1)} = -\frac{4}{6} = -\frac{2}{3}$$

(3) Consider the function $f(x) = e^{-3x} + x$.

(a) Find all critical points of the function.
(b) Use the second derivative test to attempt to classify them

a) $f'(x) = -3e^{-3x} + 1$ critical points solve $f'(x)=0$: $e^{-3x} = \frac{1}{3}$.

$-3x = \ln(1/3) = -\ln(3)$

$x = \frac{1}{3}\ln(3)$.

b) $f''(x) = 9e^{-3x}$

$f''\left(\frac{1}{3}\ln(3)\right) > 0 \Rightarrow$ local min

(4) Consider the function $f(x) = \dfrac{3x^4 - 1}{x^2}$. $= 3x^2 - x^{-2}$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all the points of inflection.
(c) Determine the intervals where $f(x)$ is concave up and concave down.

a) vertical asymptotes at $x = 0$ $\lim_{x \to \pm\infty} f(x) = +\infty$, no horizontal asymptotes.

b) $f'(x) = 6x + 2x^{-3}$

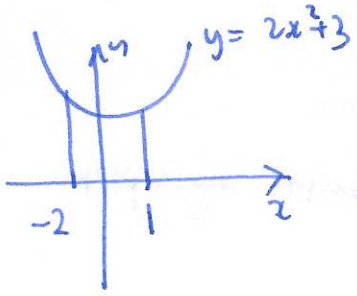
$f''(x) = 6 - 6x^{-4}$ inflection points, solve $f''(x) = 0$: $x^4 = 1$ $x = \pm 1$.

$f''(x)$ + − +

concave up $(-\infty, -1) \cup (1, \infty)$

concave down $(-1, 1)$

(5) Find the area under the graph of $y = 2x^2 + 3$ between $x = -2$ and $x = 1$.



$$\int_{-2}^{1} 2x^2 + 3\,dx = \left[\frac{2}{3}x^3 + 3x\right]_{-2}^{1}$$

$$= \frac{2}{3} + 3 - \left(\frac{2}{3} \cdot -8 - 6\right) = \frac{2 + 9 + 16 + 18}{3} = 15$$

Math 231 Calculus 1 Spring 26 Midterm 3b Part 2

Name: Solutions

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5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(6) Find the indefinite integral $\int \frac{3}{x} - 2e^x + \frac{2}{\sqrt[4]{x}}\, dx.$

$2x^{-1/4}$.

$3\ln|x| - 2e^x + \frac{8}{3}\, x^{3/4} + C$

(7) Find the indefinite integral $\int x\cos(x^2+3)\,dx$.

$u = x^2+3$

$\frac{du}{dx} = 2x$

$$\int x\cos(u)\frac{dx}{du}\,du$$

$$\int x\cos(u)\cdot\frac{1}{2x}\,du = \frac{1}{2}\int\cos(u)\,du$$

$$= \frac{1}{2}\sin(u)+c$$

$$= \frac{1}{2}\sin(x^2+3)+c$$

(8) Find the definite integral $\int_0^1 \frac{e^x}{\sqrt{3e^x+1}}\,dx$.

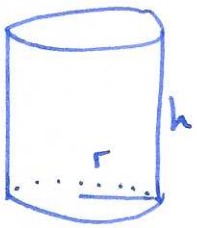
$u = 3e^x + 1$

$\frac{du}{dx} = 3e^x$

$$\int_4^{3e+1} \frac{e^x}{\sqrt{u}} \frac{dx}{du}\,du = \int_4^{3e+1} e^x u^{-1/2} \cdot \frac{1}{3e^x}\,du$$

$$= \frac{1}{3}\int_4^{3e+1} u^{-1/2}\,du = \left[\frac{2}{3}u^{1/2}\right]_4^{3e+1} = \frac{2}{3}\left(\sqrt{3e+1} - 2\right).$$

(9) A cylindrical container has a round base but no top. What dimensions minimize surface area if the total volume of the container is $8m^3$?



$V = \pi r^2 h = 8 \Rightarrow h = \frac{8}{\pi r^2}$

$A = 2\pi r h + \pi r^2$

$A = 2\pi r \cdot \frac{8}{\pi r^2} + \pi r^2 = \frac{16}{r} + \pi r^2$

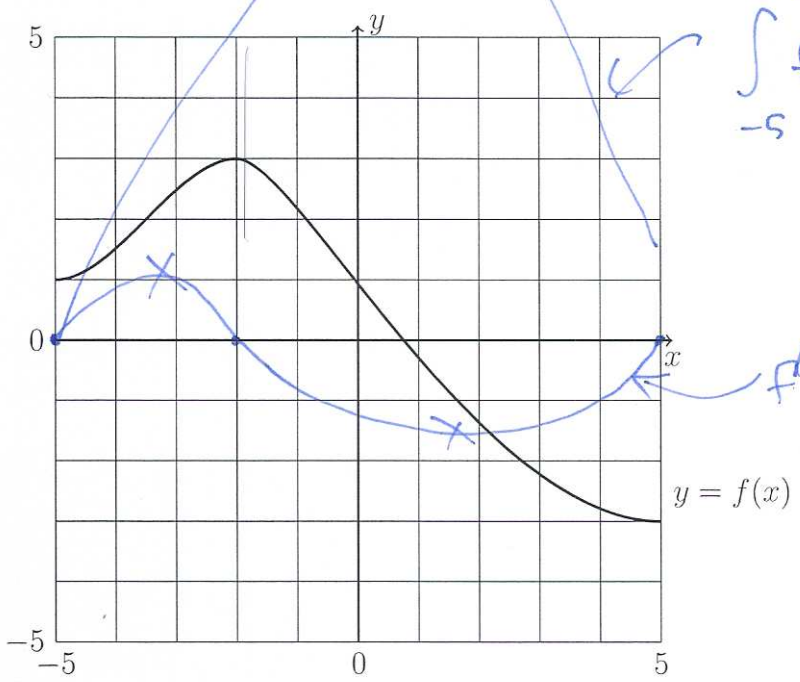
$\frac{dA}{dr} = -\frac{16}{r^2} + 2\pi r$ critical point solve $\frac{dA}{dr} = 0$:

$2\pi r = \frac{16}{r^2}$

$r^3 = \frac{8}{\pi}$ $r = \frac{2}{\sqrt[3]{\pi}}$

$h = \frac{8}{\pi \cdot \left(\frac{2}{\sqrt[3]{\pi}}\right)^2}$

(10) Consider the function $f(x)$ defined by the following graph.



(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection of $f(x)$.

(c) Sketch the graph of $\int_{-5}^{x} f(t)\ dt$.