Math 231 Calculus 1 Spring 26 Midterm 3a Part 1

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a US Letter page of notes but no calculator.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) Use l'Hôpital's rule to find $\lim_{x\to\infty} \frac{\ln(x)}{\sqrt[3]{x}}$.

l'H

$$= \lim_{x\to\infty} \frac{1/x}{\frac{1}{3}x^{-2/3}} = \lim_{x\to\infty} 3x^{-1/3} = 0$$

(2) Use l'Hôpital's rule to find $\lim_{x\to 0} \dfrac{e^{-2x^2}-1}{\cos(3x)-1}$.

$$\stackrel{l'H}{=} \lim_{x\to 0} \frac{e^{-2x^2}\cdot(-4x)}{-\sin(3x)\cdot 3} \stackrel{l'H}{=} \lim_{x\to 0} \frac{e^{-2x^2}\cdot 16x^2 + e^{-2x^2}\cdot(-4)}{-\cos(3x)\cdot 9} = \frac{4}{9}$$

(3) Consider the function $f(x) = e^{-2x} + x$.

(a) Find all critical points of the function.
(b) Use the second derivative test to attempt to classify them

a) $f'(x) = -2e^{-2x} + 1$ critical points: solve $f'(x) = 0$

$$e^{-2x} = \frac{1}{2}.$$

$$-2x = \ln(1/2) = -\ln(2)$$

$$x = \frac{1}{2}\ln(2)$$

b) $f''(x) = 4e^{-2x}$

$f''(\frac{1}{2}\ln(2)) = 4 \cdot \frac{1}{2} = 2 > 0 \Rightarrow$ local min

(4) Consider the function $f(x) = \dfrac{1-3x^4}{x^2}$. $= \dfrac{1}{x^2} - 3x^2 = x^{-2} - 3x^2$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all the points of inflection.
(c) Determine the intervals where $f(x)$ is concave up and concave down.

a) vertical asymptote at $x=0$ $\quad \lim_{x \to 0} f(x) = -\infty$ $\quad$ no horizontal asymptotes.

b) $f'(x) = -2x^{-3} - 6x$

$f''(x) = +6x^{-4} - 6$

$= 6\left(\frac{1}{x^4} - 1\right)$

inflection points solve $f''(x) = 0$: $\quad x^4 = 1$

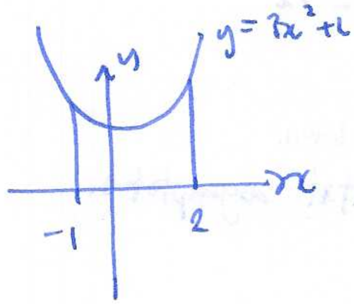
$x = \pm 1$.

$-1 \qquad +1$

$- \qquad + \qquad - \qquad f''(x)$

concave up $(-1, 1)$
concave down $(-\infty, -1) \cup (1, \infty)$

(5) Find the area under the graph of $y = 3x^2 + 2$ between $x = -1$ and $x = 2$.



$$\int_{-1}^{2} 3x^2 + 2\,dx = \left[x^3 + 2x\right]_{-1}^{2}$$

$$= 8 + 4 - (-1 - 2) = 15$$

Math 231 Calculus 1 Spring 26 Midterm 3a Part 2

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5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

$\leftarrow -2x^{-1/3}$.

(6) Find the indefinite integral $\int 2e^x - \frac{2}{x} - \frac{2}{\sqrt[3]{x}}\, dx$.

$2e^x - \ln|x| - 3x^{2/3} + c$

(7) Find the indefinite integral $\int x \sin(x^2+2)\, dx$.

$u = x^2+2$

$\frac{du}{dx} = 2x$

$$\int x \sin(u) \frac{dx}{du}\, du$$

$$= \int x \sin(u) \frac{1}{2x}\, du$$

$$= \frac{1}{2} \int \sin(u)\, du$$

$$= -\frac{1}{2}\cos(u) + c$$

$$= -\frac{1}{2}\cos(x^2+2) + c$$

(8) Find the definite integral $\int_0^1 \frac{e^x}{\sqrt{2e^x+2}}\,dx$.

$u = 2e^x + 2$

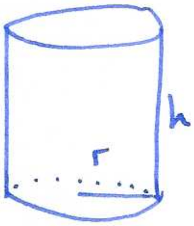
$\frac{du}{dx} = 2e^x$

$\int_4^{2e+2} \frac{e^x}{\sqrt{u}} \frac{dx}{du}\,du$

$= \int_4^{2e+2} e^x u^{-1/2} \frac{1}{2e^x}\,du = \frac{1}{2}\int_4^{2e+2} u^{-1/2}\,du = \cancel{\frac{1}{2}}\left[\cancel{2}u^{1/2}\right]_4^{2e+2}$

$= \sqrt{2e+2} - 2$

(9) A cylindrical container has a round base but no top. What dimensions minimize surface area if the total volume of the container is 8m^3?



$$V = \pi r^2 h = 8 \Rightarrow h = \frac{8}{\pi r^2}$$

$$A = 2\pi r h + \pi r^2$$

$$A = 2\pi r \cdot \frac{8}{\pi r^2} + \pi r^2 = \frac{16}{r} + \pi r^2$$

$$\frac{dA}{dr} = -\frac{16}{r^2} + 2\pi r$$

critical point solve $\frac{dA}{dr} = 0$:

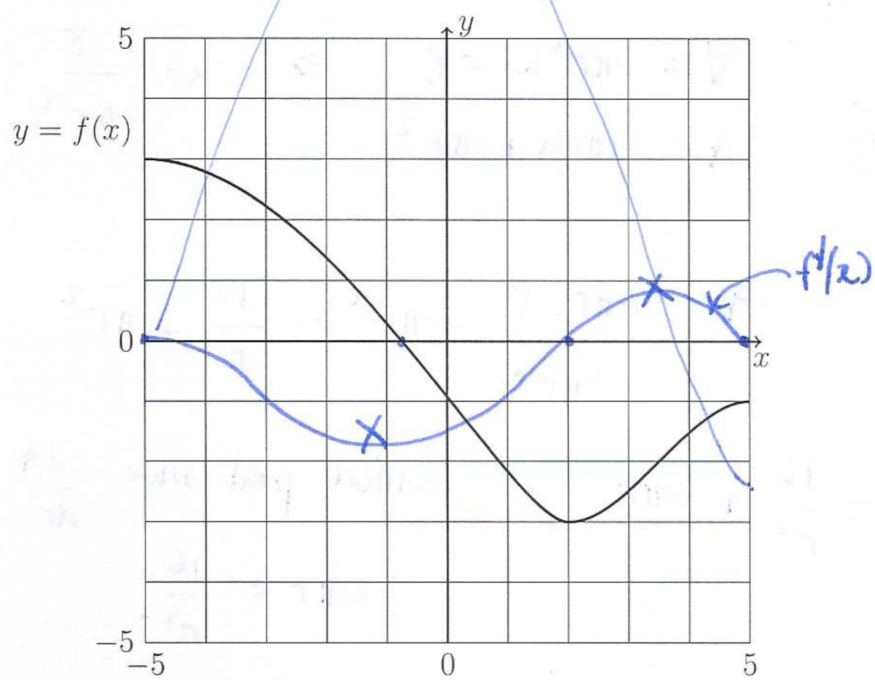
$$2\pi r = \frac{16}{r^2}.$$

$$r^3 = \frac{8}{\pi} \qquad r = \frac{2}{\sqrt[3]{\pi}}.$$

$$h = \frac{8}{\pi \cdot \left(\frac{2}{\sqrt[3]{\pi}}\right)^2}$$

$\int_{-5}^{x} f(t)\,dt$

(10) Consider the function $f(x)$ defined by the following graph.



(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection of $f(x)$. (x)

(c) Sketch the graph of $\int_{-5}^{x} f(t)\, dt$.