Math 231 Calculus 1 Fall 25 Midterm 2b Part 1

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a US letter page of notes, but no calculators.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the derivative of the following functions.

(a) $f(x) = x^2 \cos x$.

$2x \cos x + x^2 \cdot -\sin x$

(b) $f(x) = \dfrac{\ln x}{e^x}$.

$$\frac{e^x \cdot \frac{1}{x} - \ln x \cdot e^x}{e^{2x}}$$

(2) (10 points) Find the derivative of the function $f(x) = \sin^{-1}(1 + 3x^2)$.

$$\frac{1}{\sqrt{1-(1+3x^2)^1}} \cdot 6x$$

(3) (10 points) Find the second derivative of the function $f(x) = \dfrac{1}{\sqrt{4x-1}}$.

$= (4x-1)^{-1/2}$

$f'(x) = -\frac{1}{2}(4x-1)^{-3/2} \cdot 4$

$f''(x) = \frac{3}{4}(4x-1)^{-5/2} \cdot 16$

(4) (10 points) Use implicit differentiation to find the tangent line to the curve given by the equation $xy + 2xe^y = 2$ at the point $(1, 0)$.

$$y + xy' + 2e^y + 2xe^y y' = 0$$

$$y' + 2 + 2y' = 0$$

$$y' = -\frac{2}{3}$$

$$y - 0 = -\frac{2}{3}(x - 1)$$

(5) Find the following limit: $\lim_{x\to 0} \dfrac{e^{-3x^2}-1}{\cos(2x)-1}$

$$\overset{l'H}{=} \lim_{x\to 0} \frac{e^{-3x^2}\cdot(-6x)}{-\sin(2x)\cdot 2}$$

$$\overset{l'H}{=} \lim_{x\to 0} \frac{e^{-3x^2}\cdot(-6x)^2 + e^{-3x^2}\cdot(-6)}{-\cos(2x)\cdot 4} = \frac{6}{4} = \frac{3}{2}$$

Math 231 Calculus 1 Fall 25 Midterm 2b Part 2

Name: Solutions

- I will count your best 8 of the following 10 questions.
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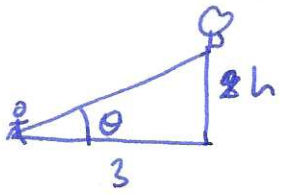
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(6) (10 points) A hot air balloon rises vertically from a point 3km away. When you see it at an angle of $\pi/6$, the angle is increasing at a rate of 0.1 radians/min. How fast is the balloon rising?



$$\tan\theta = \frac{h}{3}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{3}\frac{dh}{dt}$$

$$\sec^2\left(\frac{\pi}{6}\right) \cdot 0.1 = \frac{1}{3}\frac{dh}{dt}$$

$$\frac{1}{(\sqrt{3}/2)^2} \cdot \frac{1}{10} = \frac{1}{3}\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{3} \cdot \frac{1}{10} \cdot 3 = \frac{2}{5} \text{ km/min}$$

(7) (10 points) Use linear approximation to estimate $\sqrt[3]{63}$. Write down an expression for the percentage error in your approximation, but do not evaluate it.

$f(x) = \sqrt[3]{x} = x^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f(63) \approx f(64) + f'(64).(-1)$

$= 4 + \frac{1}{3} \cdot \frac{1}{16} \cdot -1$

$= 4 - \frac{1}{48} = 3\frac{47}{48}$

percentage error : $\dfrac{\left| 3\frac{47}{48} - \sqrt[3]{63} \right|}{\sqrt[3]{63}} \times 100$

(8) Find the critical points for the function $f(x) = 3x - x\ln(x)$ and use the second derivative test to classify them.

$f'(x) = 3 - \ln(x) - x \cdot \frac{1}{x}$

critical point. $f'(x) = 0$: $3 - \ln(x) - 1 = 0$

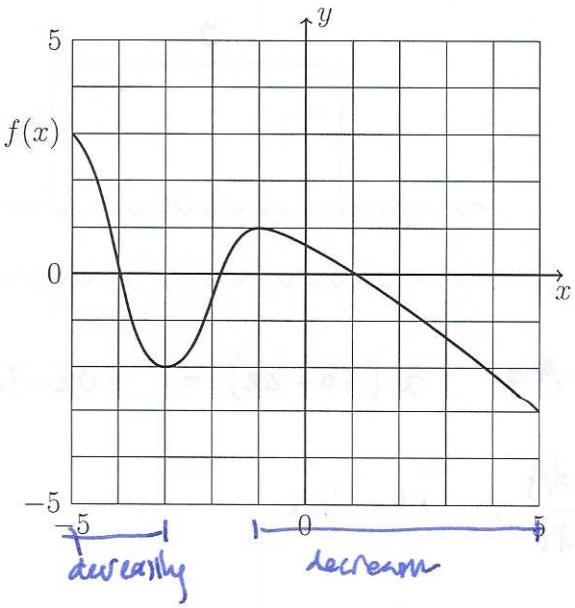
$\ln(x) = 2$

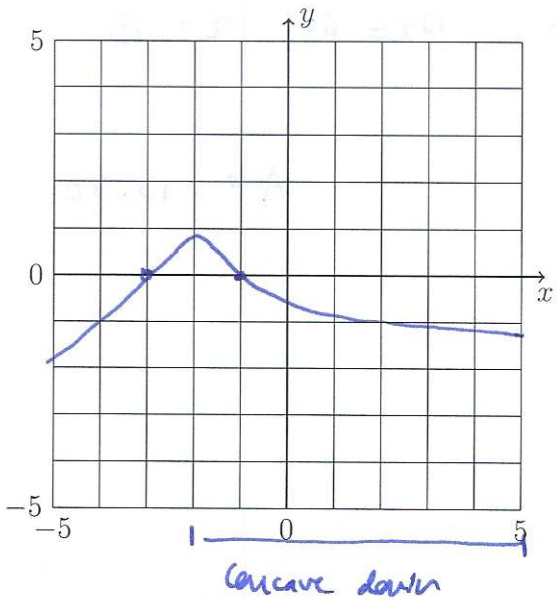
$x = e^2$

$f''(x) = -\frac{1}{x}$

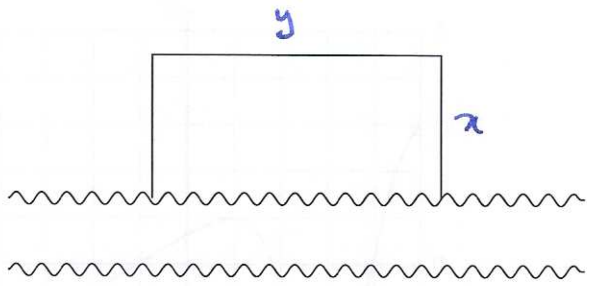
$f''(e^2) = -\frac{1}{e^2} < 0 \Rightarrow$ local max.

(9) (10 points) The graph of the function $f(x)$ is shown below. On the top set of axes mark where $f(x)$ is decreasing. On the lower set of axes sketch $f'(x)$, and then use this to find where $f(x)$ is concave down.





(10) (10 points) You wish to fence off a rectangular field beside a river and have 60m of fencing. Using the river as one side, you only need fencing for the other three sides, as shown below. What is the largest area field you can make?



$A = xy$

$L = 2x + y = 60$

$A = x(60 - 2x) = 60x - 2x^2$

$\frac{dA}{dx} = 60 - 4x$

critical point: $\frac{dA}{dx} = 0$: $4x = 60$ $x = 15$

$A = 15 \cdot 30 = 450$