

Math 231 Calculus 1 Spring 26 Midterm 1b Part 1

Name: Solutions

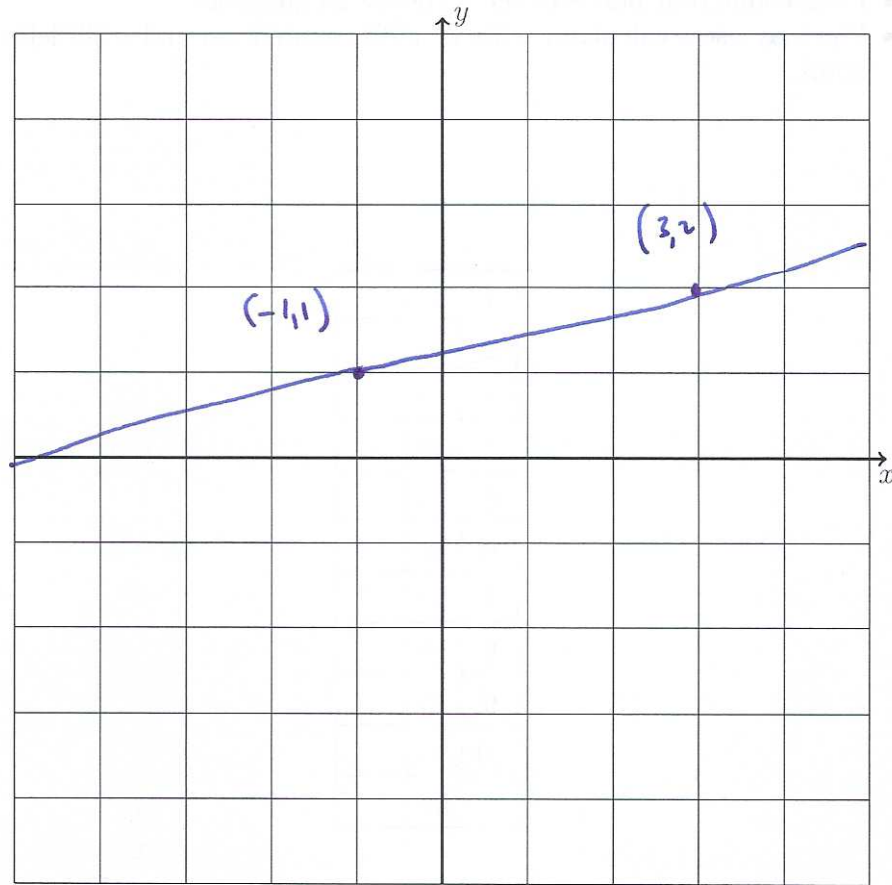
- I will count your best 8 of the following 10 questions.
- You may use a calculator without CAS capabilities, and a US letter page of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

2

- (1) (10 points) Plot the points  $(-1, 1)$  and  $(3, 2)$  on the grid below, and draw the straight line through the two points. Find the equation of the straight line.



slope  $m = \frac{2-1}{3-(-1)} = \frac{1}{4}$

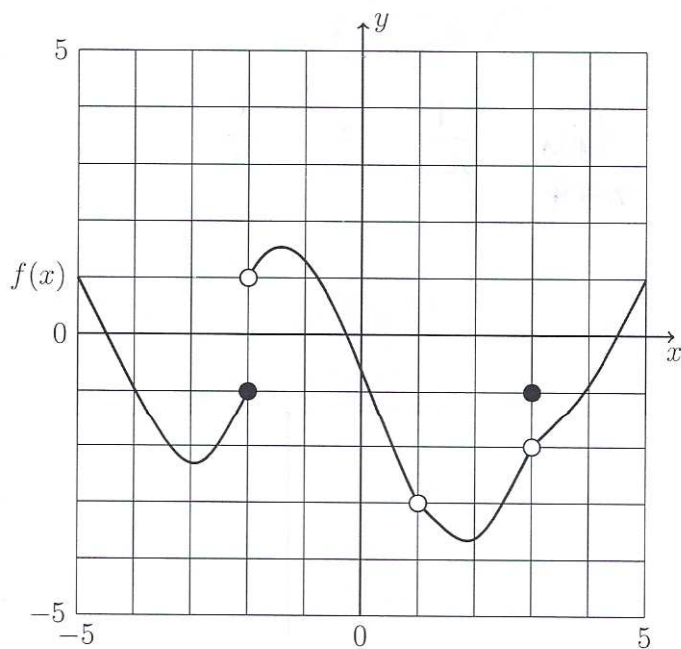
$$y - y_0 = m(x - x_0)$$

$$y - 2 = \frac{1}{4}(x - 3)$$

$$y = \frac{1}{4}x - \frac{3}{4} + 2$$

$$y = \frac{1}{4}x + \frac{5}{4}$$

- (2) (10 points) The graph of  $y = f(x)$  is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



- (a)  $\lim_{x \rightarrow -2^-} f(x)$   $-1$   
 (b)  $\lim_{x \rightarrow -2^+} f(x)$   $+1$   
 (c)  $\lim_{x \rightarrow -2} f(x)$   $DNE$   
 (d)  $\lim_{x \rightarrow 1} f(x)$   $-3$   
 (e)  $\lim_{x \rightarrow 3} f(x)$   $-2$

- (3) (10 points) Evaluate the limit algebraically. For an infinite limit, write  $+\infty$  or  $-\infty$ . If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \rightarrow 4} \frac{x-4}{x^2-6x+8}$$

$$\lim_{x \rightarrow 4} \frac{x-4}{(x-4)(x-2)} = \lim_{x \rightarrow 4} \frac{1}{x-2} = \frac{1}{2}$$

- (4) (10 points) Evaluate the limit algebraically. For an infinite limit, write  $+\infty$  or  $-\infty$ . If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \rightarrow 16} \frac{\sqrt{x} - 4}{16 - x}$$

$$\lim_{x \rightarrow 16} \frac{\sqrt{x} - 4}{(4 - \sqrt{x})(4 + \sqrt{x})} = \lim_{x \rightarrow 16} -\frac{1}{4 + \sqrt{x}} = -\frac{1}{8}$$

(5) (10 points) Use the limit definition of the derivative to differentiate  $f(x) = \frac{1}{x-1}$ .

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \frac{x-1 - (x+h-1)}{(x+h-1)(x-1)} = \lim_{h \rightarrow 0} \frac{-h}{h(x+h-1)(x-1)} \\
 &= \lim_{h \rightarrow 0} \frac{-1}{(x+h-1)(x-1)} = \frac{-1}{(x-1)^2}
 \end{aligned}$$

(6) (10 points) Find the following limit.

$$\lim_{x \rightarrow \infty} \frac{\sqrt{3+2x^4}}{4-3x^2}$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{3+2x^4}/x^2}{(4-3x^2)/x^2} = \lim_{x \rightarrow \infty} \frac{\sqrt{3/x^4+2}}{(4/x^2)-3} = -\frac{\sqrt{2}}{3}$$

$$x^4 - 4x^{1/3} + 2x^{-1/2}$$

9

(7) Find the first and second derivatives of  $f(x) = x^4 - 4\sqrt[3]{x} + \frac{2}{\sqrt{x}}$ .

$$f'(x) = 4x^3 - \frac{4}{3}x^{-2/3} - x^{-3/2}$$

$$f''(x) = 12x^2 + \frac{8}{9}x^{-5/3} + \frac{3}{2}x^{-5/2}$$

(8) Find the first and second derivatives of  $f(x) = 3 \sin(x)e^x$ .

$$f'(x) = 3\cos(x)e^x + 3\sin(x)e^x$$

$$f''(x) = -3\sin(x)e^x + 3\cos(x)e^x + 3\cos(x)e^x + 3\sin(x)e^x$$

$$= 6\cos(x)e^x$$

(9) Find the derivative of  $f(x) = \frac{3 \cos(x) - 1}{4e^x - 2}$ .

$$f'(x) = \frac{(4e^x - 2)(-3 \sin(x)) - (3 \cos(x) - 1)(4e^x)}{(4e^x - 2)^2}$$

- (10) (10 points) The graph of  $f(x)$  is given in the top picture.
- Draw in the tangent line to  $f(x)$  at  $x = 3$
  - Sketch the graph of  $f'(x)$  in the bottom picture.

