Math 231 Calculus 1 Fall 25 Midterm 1a Part 1

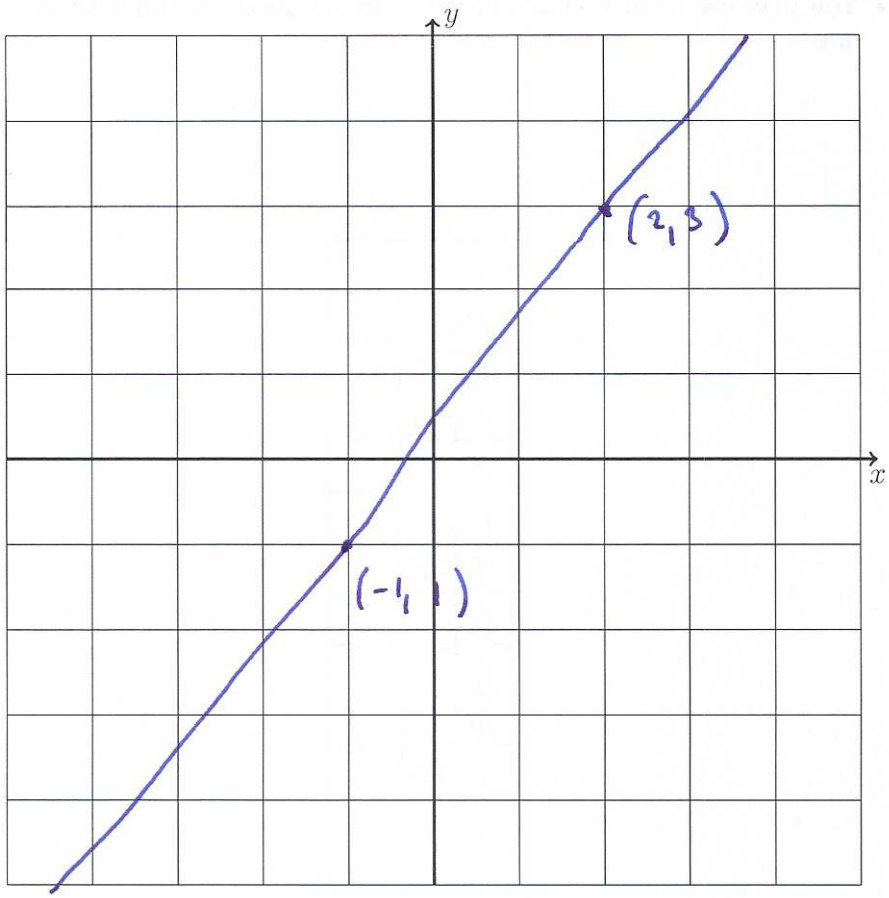
Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator without CAS capabilities, and a US letter page of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(-1,-1)$ and $(2,3)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.



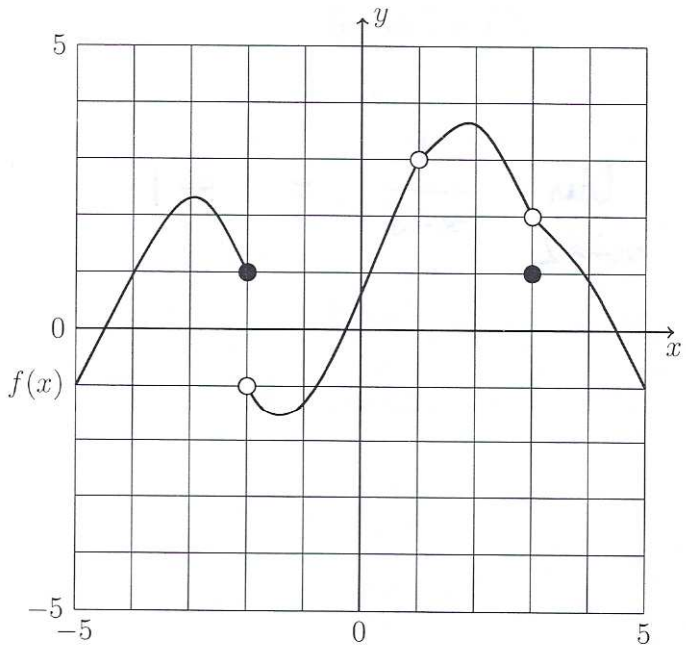
slope $m = \frac{3-(-1)}{2-(-1)} = \frac{4}{3}$

$y - 3 = \frac{4}{3}(x-2)$

$y = \frac{4}{3}x + 3 - \frac{8}{3}$

$y = \frac{4}{3}x + \frac{1}{3}$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



(a) $\lim_{x \to -2-} f(x)$ 1

(b) $\lim_{x \to -2+} f(x)$ −1

(c) $\lim_{x \to -2} f(x)$ DNE

(d) $\lim_{x \to 1} f(x)$ 3

(e) $\lim_{x \to 3} f(x)$ 2

(3) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 2} \frac{x-2}{x^2-5x+6}$$

$$\lim_{x\to 2} \frac{x-2}{(x-2)(x-3)} = \lim_{x\to 2} \frac{1}{x-3} = -1$$

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 9} \frac{3 - \sqrt{x}}{9 - x}$$

$$\lim_{x \to 9} \frac{3-\sqrt{x}}{(3-\sqrt{x})(3+\sqrt{x})} = \lim_{x \to 9} \frac{1}{3+\sqrt{x}} = \frac{1}{6}$$

(5) (10 points) Use the limit definition of the derivative to differentiate $f(x) = \dfrac{1}{x-1}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h} \frac{x-1-(x+h-1)}{(x+h-1)(x-1)} = \lim_{h\to 0} \frac{-h}{h(x+h-1)(x-1)}$$

$$= \lim_{h\to 0} \frac{-1}{(x+h-1)(x-1)} = \frac{-1}{(x-1)^2}$$

(6) (10 points) Find the following limit.

$$\lim_{x\to\infty} \frac{\sqrt{2+3x^4}}{5-x^2}$$

$$\lim_{x\to\infty} \frac{\sqrt{2+3x^4}\,/x^2}{(5-x^2)/x^2} = \lim_{x\to\infty} \frac{\sqrt{\frac{2}{x^4}+3}}{\frac{5}{x^2}+1} = \frac{\sqrt{3}}{-1} = -\sqrt{3}.$$

($\frac{2}{x^4} \to 0$, $\frac{5}{x^2} \to 0$)

$x^5 + 2x^{1/3} - 2x^{-1/2}$

(7) Find the first and second derivatives of $f(x) = x^5 + 2\sqrt[3]{x} - \frac{2}{\sqrt{x}}$.

$f'(x) = 5x^4 + \frac{2}{3}x^{-2/3} + x^{-3/2}$

$f''(x) = 20x^3 - \frac{4}{9}x^{-5/3} - \frac{3}{2}x^{-5/2}$

(8) Find the first and second derivatives of $f(x) = 2\cos(x)e^x$.

$$f'(x) = -2\sin(x)e^x + 2\cos(x)e^x$$

$$f''(x) = -2\cos(x)e^x + -2\sin(x)e^x + -2\sin(x)e^x + 2\cos(x)e^x$$

$$= -4\sin(x)e^x$$

(9) Find the derivative of $f(x) = \dfrac{3 - 2e^x}{2 - 4\sin(x)}$.

$$f'(x) = \frac{(2 - 4\sin x)(-2e^x) - (3 - 2e^x)(-4\cos(x))}{(2 - 4\sin x)^2}.$$

(10) (10 points) The graph of $f(x)$ is given in the top picture.

(a) Draw in the tangent line to $f(x)$ at $x = 1$

(b) Sketch the graph of $f'(x)$ in the bottom picture.