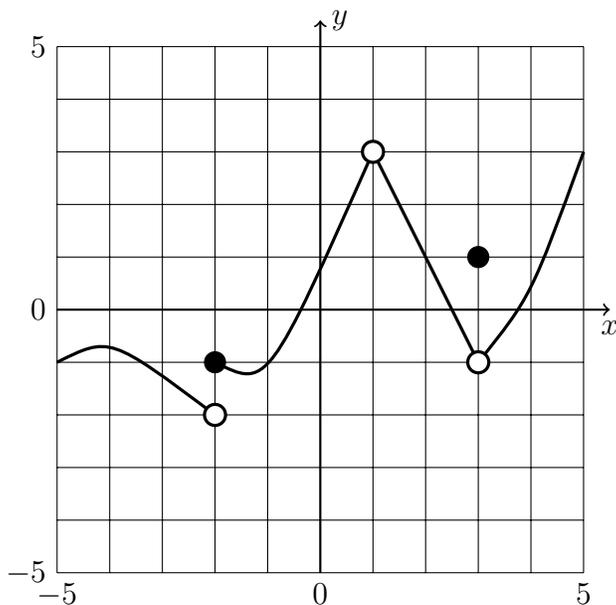


### Math 231 Calculus 1 Spring 26 Sample Midterm 1

- (1) The graph of  $y = f(x)$  is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



- (a)  $\lim_{x \rightarrow -2^-} f(x)$   
(b)  $\lim_{x \rightarrow -2^+} f(x)$   
(c)  $\lim_{x \rightarrow -2} f(x)$   
(d)  $\lim_{x \rightarrow 1} f(x)$   
(e)  $\lim_{x \rightarrow 3} f(x)$

- (2) Evaluate these limits. For an infinite limit, write  $+\infty$  or  $-\infty$ . If a limit does not exist (DNE), you must justify why this is the case.

(a)  $\lim_{x \rightarrow 3} \frac{x - 3}{x^2 - x - 6}$

(b)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 - 2}}{2x - 1}$

(c)  $\lim_{x \rightarrow 0} \frac{\sin 2x}{3x}$

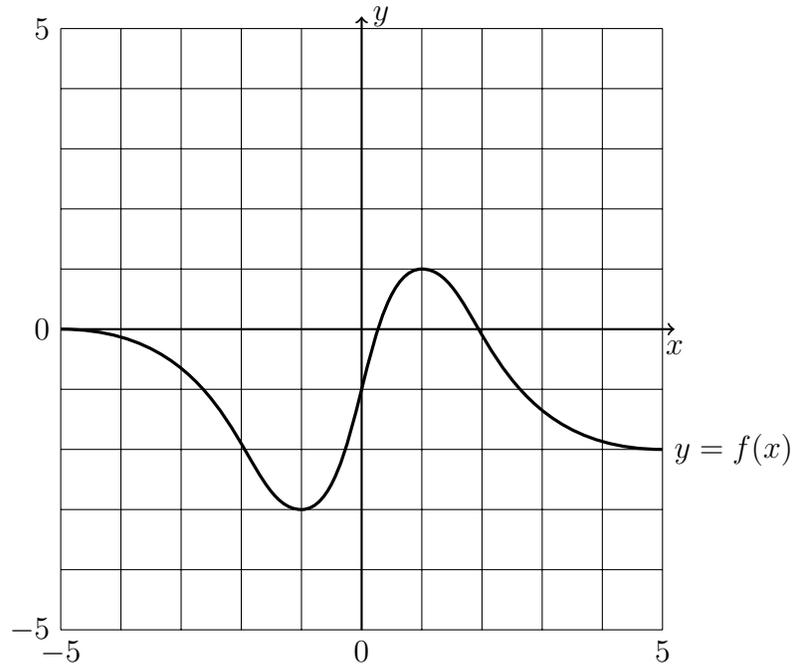
(d)  $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x + 4} - 2}$

- (3) For what value of  $c$  (if any) is the function  $f(x)$  continuous at  $x = 2$ ? Justify your answer.

$$f(x) = \begin{cases} \frac{1}{x-1} & x < 2 \\ c & x = 2 \\ \frac{2 \cos(\pi x)}{x} & x > 2 \end{cases}$$

- (4) For a sphere of radius  $r$ , its volume is  $V = \frac{4}{3}\pi r^3$ . What is the average rate of change of volume when the radius increases from  $r = 2$  to  $r = 3$ ?
- (5) Show that  $e^x = 4 - x$  has a solution for some  $x > 0$ . You do not need to find this solution.

(6) Consider the function  $f(x)$  defined by the following graph.



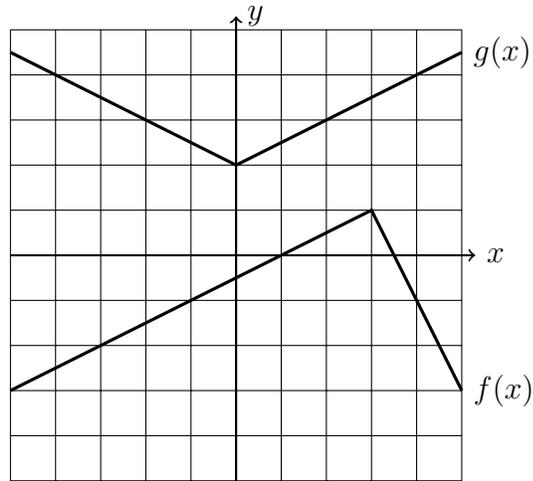
- (a) Label all regions where  $f'(x) < 0$ .  
 (b) Label all regions where  $f'(x) > 0$ .  
 (c) Sketch a graph of  $f'(x)$  on the figure.  
 (d) What is  $\lim_{x \rightarrow \infty} f(x)$ ?  
 (e) What is  $\lim_{x \rightarrow -\infty} f'(x)$ ?
- (7) Use the limit definition of the derivative to evaluate  $f'(2)$ , where
- (a)  $f(x) = x^2 + x$   
 (b)  $f(x) = \frac{1}{x+1}$   
 (c)  $f(x) = \sqrt{1+x}$
- (8) Find the derivatives of the following functions

- |                                     |                                         |
|-------------------------------------|-----------------------------------------|
| (a) $4x^5 - 2x^3$                   | (e) $\sin^2(x)$                         |
| (b) $\sqrt{x^5} - 2\sqrt[4]{1/x^3}$ | (f) $\frac{4 - \sqrt{2x}}{3 - \sin(x)}$ |
| (c) $2x^4 e^x$                      | (g) $x^3 e^x \cos(x)$                   |
| (d) $\frac{5x - 2}{2 - 4x}$         |                                         |

4

(9) Find the second derivatives of the functions from Q8abce.

(10) The graphs of the functions  $f$  and  $g$  are shown below.



- (a) Let  $h(x) = f(x)g(x)$ . Find  $h'(-1)$ .
- (b) Let  $h(x) = f(x)/g(x)$ . Find  $h'(2)$ .